CHAPTER THREE: DESIGN *OF* AXIALLY AND ECCENTRICALLY LOADED COLUMN

3.1 Introduction

A column is a vertical structural member transmitting axial compression loads with or without moments. **The cross sectional dimensions** of a column are generally considerably less than its height. Column support mainly vertical loads from the floors and roof and transmit these loads to the foundation

In **construction**, the reinforcement and concrete for the beam and slabs in a floor are place once the concrete has hardened; the reinforcement and concrete for the columns over that floor are placed followed by the next higher floor.

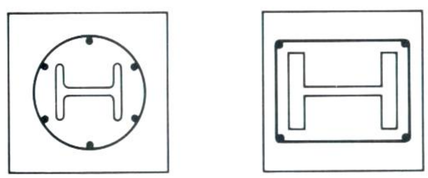
3.2. Columns may be classified based on the following criteria:

a) ***Classification on the basis of geometry****; rectangular, square, circular, L-shaped, T-shaped, etc. depending on the structural or architectural requirements.*
b) ***Classification on the basis of composition;*** *composite columns, Infilled columns, etc.*
c) ***Classification on the basis of lateral reinforcement****; tied columns, spiral columns.*
d) ***Classification on the basis of manner by which lateral stability*** *is provided to the structure as a whole; braced columns, unbraced columns.*
e) ***Classification on the basis of sensitivity to second order effect due to lateral displacements; sway columns****, non-sway columns.*
f) ***Classification on the basis of degree of slenderness****; short column, slender column.*
g) ***Classification on the basis of loading****: axially loaded column, columns under uni-axial bending, columns under biaxial bending.*

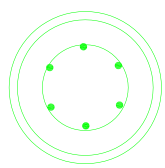
1) Classification on the basis of composition:

Composite/Infilled Columns

a) **Composite columns**: columns in which steel structural members are encased in a concrete. Main reinforcement bars positioned with ties or spirals are placed around the structural member



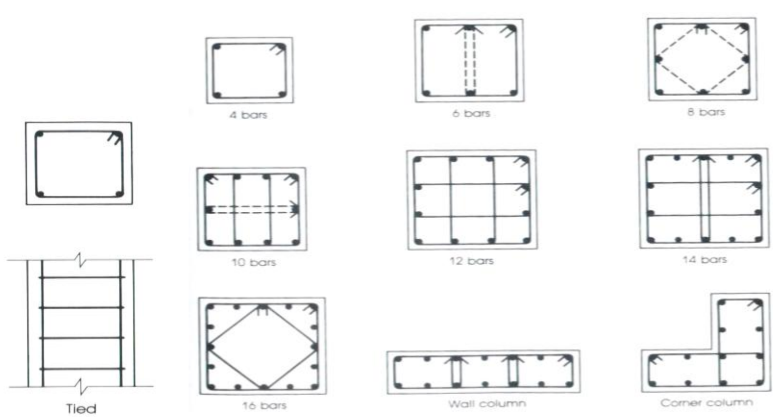
b) **Infilled columns:** columns having steel pipes filled with plain concrete or lightly reinforced concrete.



2) Classification on the basis of lateral reinforcement

Tied/Spiral Columns

a) **Tied columns**: columns where main (longitudinal) reinforcements are held in position by separate ties spaced at equal intervals along the length). Tied columns may be, square, rectangular, L-shaped, circular or any other required shape. And over 95% of all columns in buildings in non-seismic regions are tied columns.



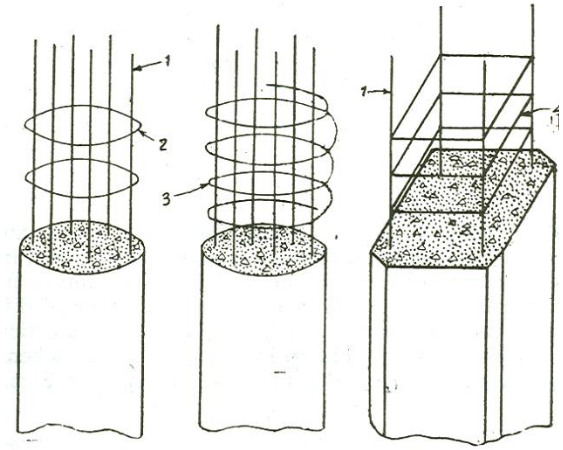


Fig.3.1 Tied columns and its typical arrangement

b) **Spiral columns**: columns which are usually circular in cross section and longitudinal bars are wrapped by a closely spaced spiral.

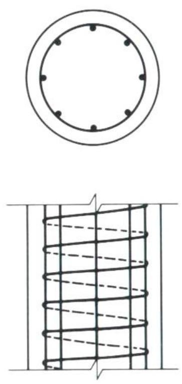


Fig. 3.2 Spiral column

Behavior of Tied and Spiral columns:

The load deflection diagrams (see Fig. 3.3) show the behavior of tied and spiral columns subjected to axial load.

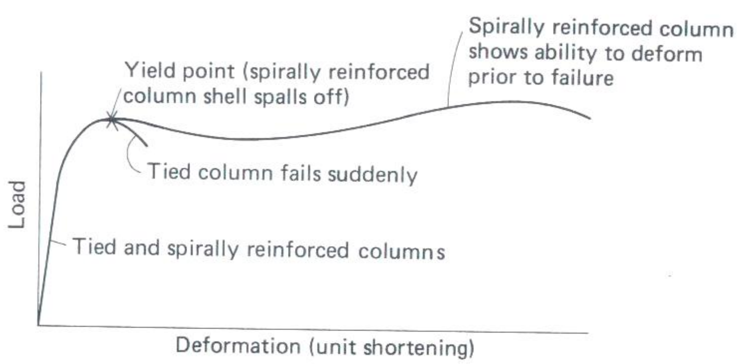


Fig. 3.3 load deflection behavior of tied and spiral columns

The initial parts of these diagrams are similar. As the maximum load is reached vertical cracks and crushing develops in the concrete shell outside the ties or spirals, and this concrete spalls off. When this happens in a tied column, the capacity of the core that remains is less than the load and the concrete core crushes and the reinforcement buckles outward between the ties. This occurs suddenly, without warning, in a brittle manner.

When the shell spalls off in spiral columns, the column doesn't fail immediately because the strength of the core has been enhanced by the tri axial stress resulting from the confinement of the core by the spiral reinforcement. As a result the column can undergo large deformations before collapses (yielding of spirals). Such failure is more ductile and gives warning to the impending failure.

Accordingly, ductility in columns can be ensured by providing spirals or closely spaced ties.

3) Classification on the basis of manner by which lateral stability: Braced/Unbraced Columns

a) Unbraced columns

An unbraced structure is one in which frames action is used to resist horizontal loads. In such a structure, the horizontal loads are transmitted to the foundations through bending action in the beams and columns. The moments in the columns due to this bending can substantially reduce their axial (vertical) load carrying capacity. Unbraced structures are generally quit flexible and allow horizontal displacement (see Fig.3.4). When this displacement is sufficiently large to influence significantly the column moments, the structure is termed a sway frame.

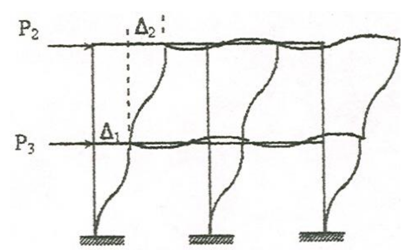


Fig. 3.4 Sway frame/unbraced columns

b) Braced columns:

Although, fully non sway structures are difficult to achieve in practice, EBCS-2 or EC-2 allows a structure to be classified as non sway if it is braced against lateral loads using substantial bracing members such as ***shear walls, elevators, stairwell shafts, diagonal bracings or a combination of these*** (See Fig.3.5). A column with in such a non sway structure is considered to be ***braced*** and the second order moment on such column $P-\Delta$ is negligible. This may be assumed to be the case if the frame attracts not more than 10% of the horizontal loads.

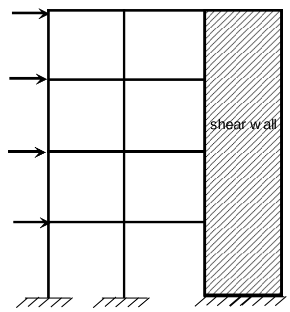


Fig. 3.5 Non-sway frame/braced columns

4) Classification on the basis of sensitivity to second order effect due to lateral displacements: Sway/Non-Sway Columns

a) Sway frame

A frame may be considered as sway if the effects of horizontal displacements of its nodes $(P-\Delta)$ are significant to be considered in design. In sway frame, the bending moment the column is increased by an additional amount$P\Delta$, where P is the axial force and Δ is the relative displacements of the ends of the column (see Fig. 3.4). Thus to maximize the axial load capacity of columns, non sway structures should be used whenever possible.

b) Non-sway frame

A frame may be considered as non-sway if its response to in-plane horizontal forces is sufficiently stiff so that the additional internal forces or moments arising from horizontal displacements of its nodes can be neglected in design.

A frame may be classified as non-sway for a given load case if the critical load ratio for that load case satisfies the criterion:

$$\frac{N_{sd}}{N_{cr}} \leq 0.1$$

Where: N_{sd} is the design value of the total vertical load

N_{cr} is its critical value for failure in a sway mode

In Beam-and-column type plane frames in building structures with beams connecting each column at each story level may be classified as non-sway for a given load case, **when first-order theory is used, the horizontal displacements in each story due to the design loads (both horizontal and vertical), plus the initial sway imperfection satisfy the following criteria.**

$$\boldsymbol{\frac{N\delta}{HL} \leq 0.1}$$

Where:

- δ is the horizontal displacement at the top of the story, relative to the bottom of the story
- L is the story height
- H is the total horizontal reaction at the bottom of the story
- N is the total vertical reaction at the bottom of the story,

For frame structures, the effects of imperfections may be allowed for in frame analysis by means of an equivalent geometric imperfection in the form of an initial sway imperfection (assuming that the structure is inclined to the vertical at an angle) ϕ determined by:

a) For single storey frames or for structures loaded mainly at the top

$$\tan \Phi = \frac{1}{150}$$

b) For other types of frames

$$\tan \Phi = \frac{1}{200}$$

Where the effects of imperfections are smaller than the effects of design horizontal actions, their influence may be ignored. Imperfections need net be considered in accidental combinations of actions.

The displacement δ in the above equation shall be determined using stiffness values for beams and columns corresponding to the ultimate limit state. As an approximation, displacements calculated using moment of inertia of the gross section may be multiplied by the ratio of the gross column stiffness I_g to the effective column stiffness I_e (see the following section) to obtain δ.

All frames including sway frames shall also be checked for adequate resistance to failure in non-sway modes

Determination of storey Buckling Load $\mathbf{N_{cr}}$

Unless more accurate methods are used, the buckling load of a story may be assumed to be equal to that of the substitute beam-column frame defined in Fig.3.6 and may be determined as:

$$N_{cr} = \frac{\pi^2 \, EI_e}{L_e^2}$$

Where:

- EI_e is the effective stiffness of the substitute column designed using the equivalent reinforcement area.
- $\mathrm{L_e}$ is the effective length. It may be determined using the stiffness properties of the gross concrete section for both beams and columns of the substitute frame (see Fig.3.6b)

In lieu of a more accurate determination, the effective stiffness of a column EI_e may be taken as:

$$EI_e = 0.2E_cI_c + E_sI_s$$

Where:

- $\mathrm{E_c = 1100f_{cd}}$
- $\mathrm{E_s}$ is the modulus of elasticity of steel
- $\mathrm{I_c, I_s}$ are the moments of inertia of the concrete and reinforcement sections, respectively, of the substitute column, with respect to the centroid of the concrete section (see Fig. 3.6c) or alternatively

$$\mathrm{EI_e = \frac{M_b}{(1/r_b)} \geq 0.4E_cI_c}$$

Where: $\mathrm{M_b}$ is the balanced moment capacity of the substitute column,

$1/\mathrm{r_b}$ is the curvature at balanced load and may be taken as

$$\boldsymbol{1/r_b} = \left(\frac{5}{\mathrm{d}}\right)10^{-3}$$

The equivalent reinforcement areas, $\boldsymbol{A_{s,tot}}$ in the substitute column to be used for calculating $\boldsymbol{I_s}$and $\boldsymbol{M_b}$ may be obtained by designing the substitute column at each floor level to carry the story design axial load and amplified sway moment at the critical section. The equivalent column dimensions of the substitute column may be taken as shown in Fig, 2.6c below, in the case of rectangular columns. Circular columns may be replaced by square columns of the same cross-sectional area. In the above, concrete cover and bar arrangement in the substitute columns shall be taken to be the same as those of the actual columns.

The amplified sway moment, to be used for the design of the substitute column, may be found iteratively taking the first-order design moment in the substitute column as an initial value.

In lieu of more accurate determination, the first-order design moment, $\boldsymbol{M_{dl}}$, at the critical section of the substitute column may be determined using:

$$\mathrm{M_{dl}} = \frac{\alpha_2 + 3}{\alpha_1 + \alpha_2 + 6} HL$$

Where α_1 and α_2 are defined before and shall not exceed 10.

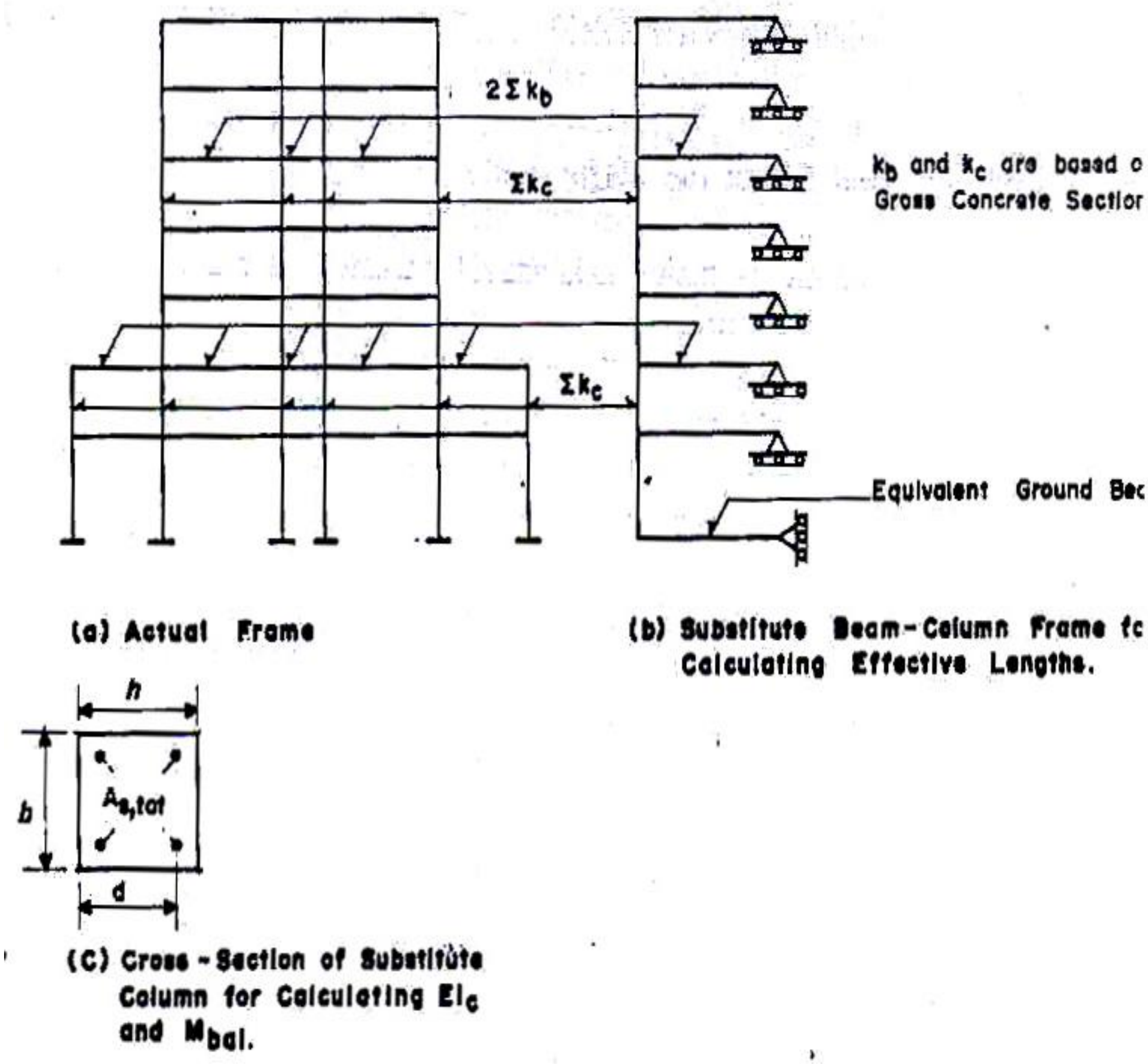


Fig. 3.6 Substitute multi storey beam-column frame

5) Classification on the basis of degree of slenderness: Short/Slender Columns

a) Short columns

They are columns with low slenderness ratio and their strengths are governed by the strength of the materials and the geometry of the cross section.

b) Slender columns

They are columns with high slenderness ratio and their strength may be significantly reduced by lateral deflection.

When an unbalanced moment or as moment due to eccentric loading is applied to a column, the member responds by bending as shown in Fig.2.7 below. If the deflection at the centre of the member is,d, then at the centre there is a force P and a total moment of $M + P\delta$. The second order bending component, $P\delta$, is due to the extra eccentricity of the axial load which results from the deflection. If the column is short δ is small and this second order moment is negligible. If on the other hand, the column is long and slender, δ is large and $P\delta$ must be calculated and added to the applied moment M.

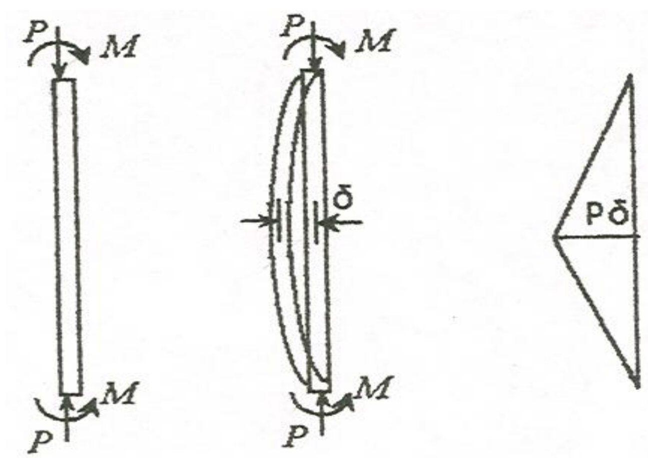


Fig 3.7

Slenderness Ratio

The significance of $P\delta$ (i.e. whether a column is short or slender) is defined by a slenderness ratio.

In EBCS 2, the slenderness ratio is defined as follows:

a) For isolated columns, the slenderness ratio is defined by:

$$\lambda = \frac{L_e}{i}$$

Where:

- L_e is the effective buckling length
- i is the minimum radius of gyration. The radius of gyration is equal to

$$i = \sqrt{\frac{I}{A}}$$

Where: I is the second moment of area of the section

A is cross sectional area

b) For multistory sway frames comprising rectangular sub frames, the following expression may be used to calculate the slenderness ratio of the columns in the same story.

$$\lambda = \sqrt{\frac{12A}{k_i L}}$$

Where: A is the sum of the cross-sectional areas of all the columns of the story

k_i is the total lateral stiffness of the columns of the story (story rigidity), with modulus of elasticity taken as unity

L is the story height

Limits of Slenderness

- The slenderness ratio of concrete columns shall not exceed 140
- Second order moment in a column can be ignored if
  a) For sway frames, the greater of

$$\lambda \leq 25$$

$$\lambda \leq \frac{15}{\sqrt{\nu_d}}$$

Where

$$\nu_d = \frac{N_{sd}}{f_{cd} A_c}$$

b) For non-sway frames

$$\lambda \leq 50 - 25\frac{M_1}{M_2}$$

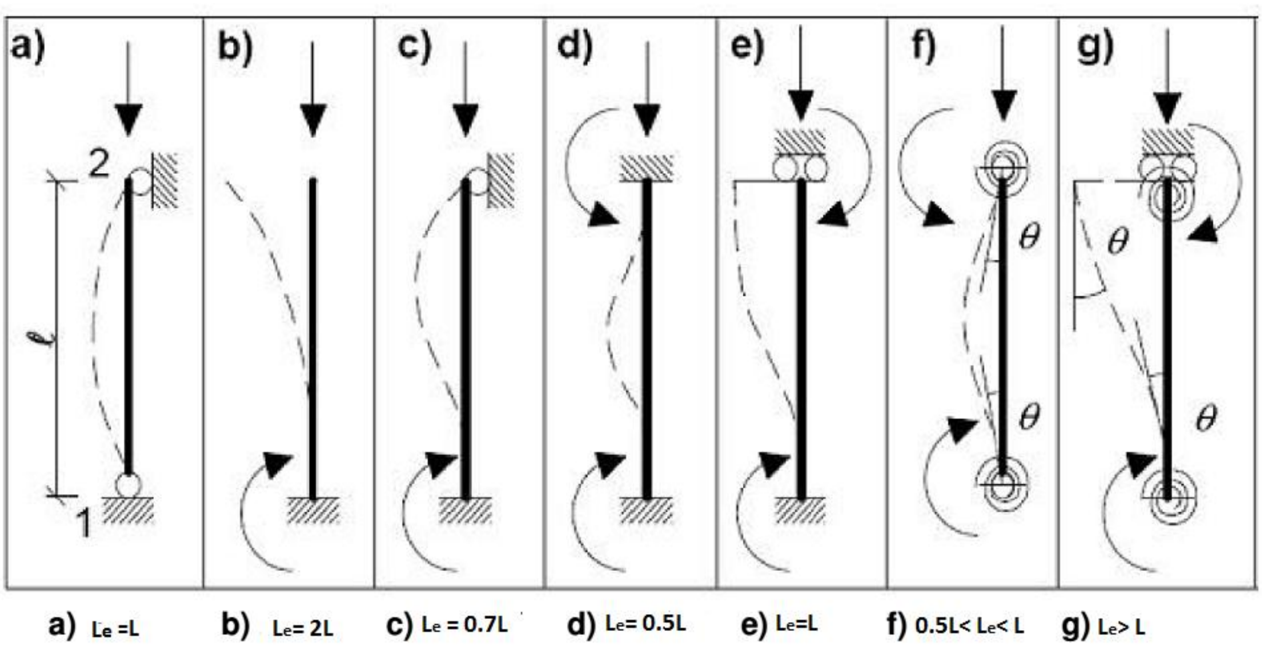
Where M_1 and M_2 are the first-order (calculated) moments at the ends, M_2 being always positive and greater in magnitude thanM_1, and M_2 being positive if member is bent in single curvature and negative if bent in double curvature

Effective Length of Columns

Effective buckling length is the length between points of inflection of columns and it is the length which is effective against buckling. The greater the effective length, the more likely the column is to be buckle.

The effective height (length) of the column, L_e***, is the distance between the two consecutive points of contra flexure or zero bending moments.*** The figure 3.8 shown below.

a) Figure 3.8 is used when the support conditions of the column can be closely represented by those shown in the figure below.



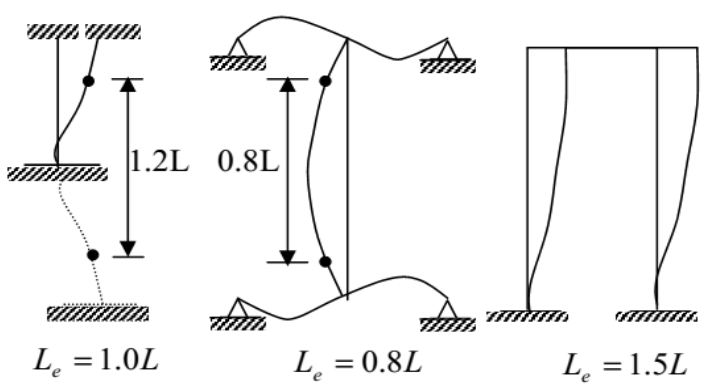


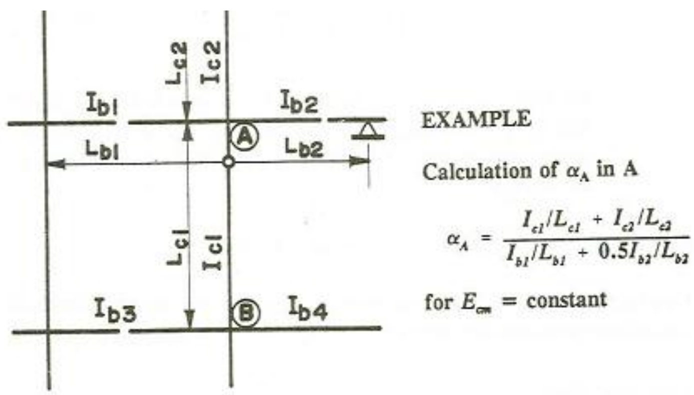
Fig 3.8: Examples of different buckling modes and corresponding effective length for isolated members

b) The effect of end restrained is quantified by the two end restrain factors α_1 and α_2

$$\alpha_1 (or\ \alpha_2) = \frac{\sum \mathrm{E_{cm}I_{col}}/L_{col}}{\sum \mathrm{E_{cm}\beta\ I_b}/L_b}$$

Where

- $\mathrm{E_{cm}}$ is modulus of elasticity of concrete
- $\mathrm{L_{col}}$ is column height
- $\mathrm{L_b}$ is span of the beam
- $\mathrm{I_{col}}, \mathrm{I_b}$ are moment of inertia of the column and beam respectively
- β is factor taking in to account the condition of restraint of the beam at the opposite end
- $\beta = 1.0$ opposite end elastically or rigidly restrained
- $\beta = 0.5$ opposite end free to rotate
- $\beta = 0$ for cantilever beam



Note that: if the end of the column is fixed, the theoretical value of $is\ 0$, but an α value of 1 is recommended for use. On the other hand, if the end of the member is pinned, the theoretical value of α is infinity, but an α value of 10 is recommended for use. The rationale behind the foregoing recommendations is that no support in reality can be truly fixed or pinned.

c) However in accordance with EBCS-2, 1995, the effective length L_e for an Reinforced concrete Column is given as,

a. Non-sway mode

$$\frac{L_e}{L} = \frac{\alpha_m + 0.4}{\alpha_m + 0.8} \geq 0.7$$

b. Sway mode

$$\frac{L_e}{L} = \sqrt{\frac{7.5+4(\alpha_1+\alpha_2)+1.6\alpha_1\alpha_2}{7.5+\alpha_1+\alpha_2}} \geq 1.15$$

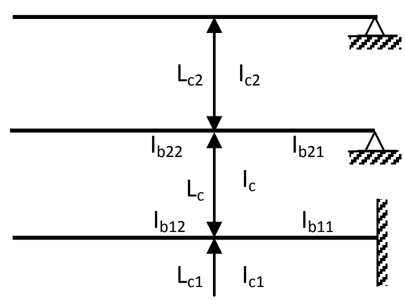
Or Conservatively $\frac{L_e}{L} = \sqrt{1 + 0.8\alpha_m} \geq 1.15$

For the theoretical model shown below.

$$\alpha_1 = \frac{k_1 + k_c}{k_{11} + k_{12}}$$

$$\alpha_2 = \frac{k_2 + k_c}{k_{21} + k_{22}}$$

$$\alpha_m = \frac{\alpha_1 + \alpha_2}{2}$$



Where

- K_1 and K_2 are column stiffness coefficients (EI/L) for the lower and the upper column respectively.
- K_c is the stiffness coefficient (EI/L) of the column being designed.
- $K_{ij} is\ the\ effective\ beam\ stiffness\ coefficient\ (EI/L)$

$$= 1.0(EI/L) for\ opposite\ end\ elastically\ or\ rigidly\ restrained.$$
$$= 0.5(EI/L)\ for\ opposite\ end\ free\ to\ rotate.$$
$$= 0.0(EI/L) for\ a\ cantilever\ beam.$$

Note that: for flats slab construction, an equivalent beam shall be taken as having the width and thickness of the slab forming the column strip.

3.3. Reinforcement arrangement and minimum requirement

Main or Longitudinal Reinforcement:

- The area of longitudinal reinforcement shall neither be less than $0.008A_c$nor more than $0.08A_c$. The upper limit shall be observed even where bars overlap.

  (Area of longitudinal reinforcement, A_s

  $0.008A_c \leq A_s \leq 0.08A_c \; or \; 0.008 \leq \frac{A_s}{A_c} \leq 0.08$)
- For columns with a larger cross-section than required by considerations of loading, a reduced effective area not less than one-half die total area may be used to determine minimum reinforcement and design strength
- The minimum number of longitudinal reinforcing bars shall be 6 for bars in a circular arrangement and 4 for bars in a rectangular arrangement
- The diameter of longitudinal bars shall not be less than 12 mm , $\Phi_l \geq 12$
- The minimum lateral dimension of a column shall be at least 150 mm and
- The minimum diameter of a spiral column is 200mm.
- The Minimum cover to reinforcement should never be less than

  (a) $\Phi \; or \; \Phi_n (\leq 40mm), or$

  (b) $(\Phi + 5\text{mm})\text{or} \; (\Phi_n + 5mm) \; if \; \text{d}_g > 32mm.$

  Where

  - Φ is the diameter of bar
  - Φ_n is the equivalent diameter for a bundle
  - $\text{d}_g$the largest nominal maximum aggregate size.

  ✓ A minimum concrete cover shall be provided in order to ensure:

  - The safe transmission of bond forces
  - That spalling will not occur
  - An adequate fire resistance
  - The protection of the steel against corrosion

Functions of Lateral Reinforcement

- They hold the longitudinal bars in position in the forms while the concrete is being placed

- They prevent the slender longitudinal bars from buckling out ward by bursting the thin concrete cover.
- **Rules for the arrangement:**

a) The diameter of ties or spirals Φ_t, shall not be less than 6 mm or one quarter of the diameter of the longitudinal bars. or

$$Diameter\ of\ ties, \Phi_t \geq \begin{cases} \frac{\Phi_l}{4} \\ 6mm \end{cases}$$

b) The center-to-center spacing of lateral reinforcement shall not exceed:

$$C/C \quad spacing \leq \begin{cases} 12\Phi_l \\ b \\ 300mm \end{cases}$$

Where:

- Φ_l *is* the minimum diameter of longitudinal bars.
- Φ_t is ties or spirals diameter / lateral reinforcement
- $C/C \ spacing \ is \ center - to - center \ spacing$
- b is least dimension of column
- 300 mm

c) Pitch of spiral $\leq 100mm$

d) Ties shall be arranged such that every bar or group of bars placed in a corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie with an included angle of not more than 135^0 and no bar shall be further than 150 mm clear on each side along the tie from such a laterally supported bar (see Fig.3.10)

e) Up to five longitudinal bars in each corner may be secured against lateral buckling by means of the main ties. The center-to-center distance between the outermost of these bars and the corner bar shall not exceed 15 times the diameter of the tie (see Fig.3.10)

$$S_{max} = 350mm$$

f) Spirals or circular ties may be used for longitudinal bars located around the perimeter of a circle. The pitch of spirals shall not exceed 100 mm.

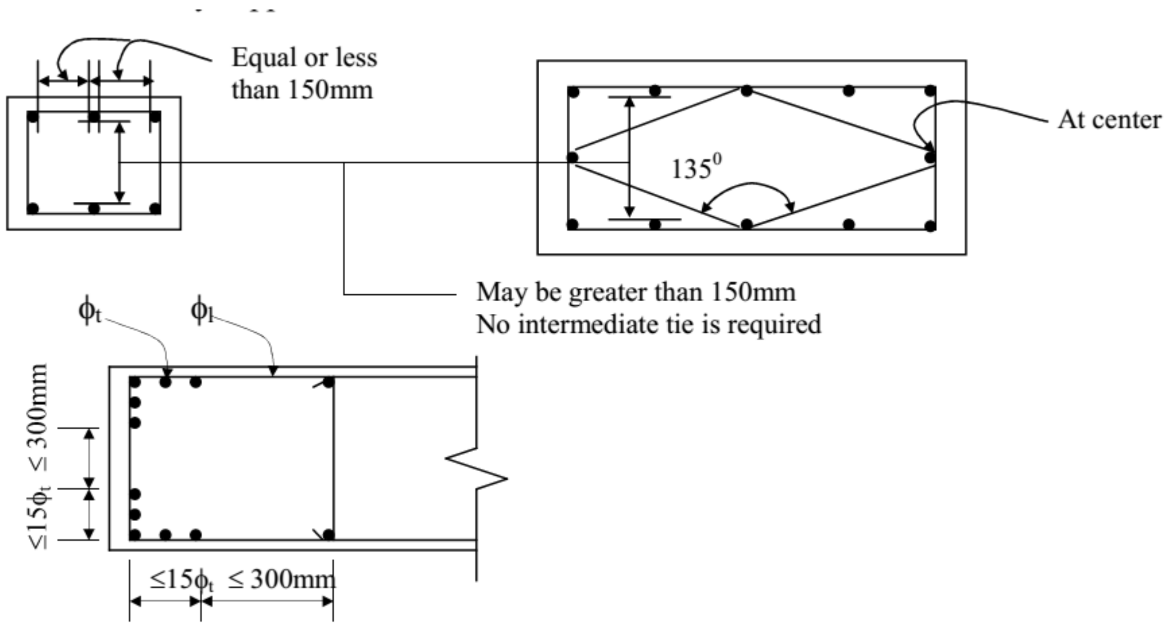


Fig. 2.10 Measurements between laterally supported column bars & Requirements for Main and Intermediate Ties

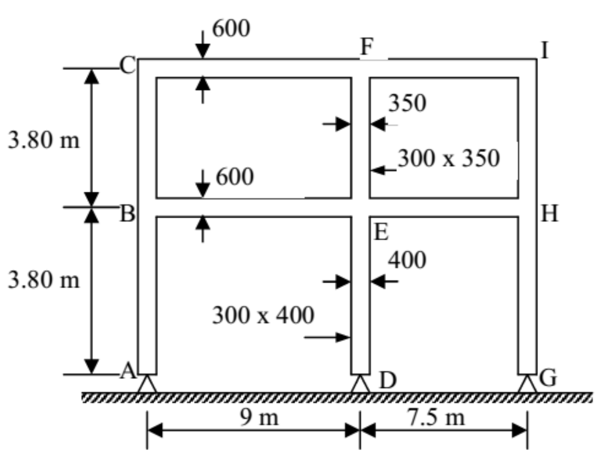
Where:

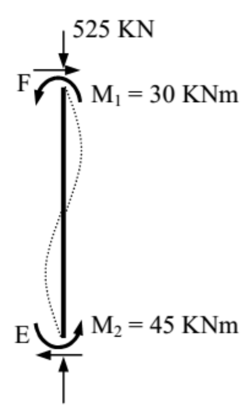
$$\Phi_l = Longituinal\ bars$$

$$\Phi_t = Main\ ties$$

Example1. (Classification of columns)

The frame shown in figure below is composed of members with rectangular cross sections. All members are constructed of the same strength concrete (E is the same for both beams and columns). Considering bending in the plane of the frame only, **classify column EF as long or short if the frame** is ***(a)*** braced and ***(b)*** unbraced. All girders are 300 x 600 mm.





Solution:

- **Moments of inertia**

Girders: $I_g = \dfrac{300x600^3}{12} = 54x10^8 mm^4$

Columns: $I_{DE} = \dfrac{300x400^3}{12} = 16x10^8 mm^4$

$$I_{EF} = \frac{300x350^3}{12} = 10.71875x10^8 mm^4.$$

- **Stiffness Coefficients:**

$$Girders: K_g = \frac{EI_g}{L_g} \Rightarrow \left[\begin{array}{l} K_{BE} = K_{CF} = \dfrac{(E)(54x10^8)}{9000} = 6x10^5 E. \\ K_{EH} = K_{FI} = \dfrac{(E)(54x10^8)}{7500} = 7.2x10^5 E. \end{array} \right.$$

Columns: $K_c = \dfrac{EI_c}{L_c} \Rightarrow \left[\begin{array}{l} K_{DE} = \dfrac{(E)(16x10^8)}{3.8x10^3} = 4.21x10^5 E \\ K_{EF} = \dfrac{(E)(10.71875)(10^8)}{3.8x10^3} = 2.82x10^5 E \end{array} \right.$

The column being considered is columnEF. Rotational stiffness at joints E and F.

$$\alpha = \frac{\Sigma(EI_{col}/L)}{\Sigma(\alpha_f EI_g/L_{eff})} = \frac{\Sigma(I_{col}/L)}{\Sigma(\alpha_f I_g/L_{eff})}$$

Joint E: $\alpha_E = \dfrac{4.21x10^5 + 2.82x10^5}{6x10^5 + 7.2x10^5} = 0.53$

Joint F: $\alpha_F = \dfrac{2.82x10^5}{6x10^5 + 7.2x10^5} = 0.21$

$$\alpha_m = \frac{\alpha_E + \alpha_F}{2} = \frac{0.53 + 0.21}{2} = 0.37$$

(a) For a braced column (Non sway structure) for design

$$\frac{L_e}{L} = \frac{\alpha_m + 0.4}{\alpha_m + 0.8} = \frac{0.37 + 0.4}{0.37 + 0.8} = 0.66 \geq 0.7$$

$$L_e = (0.7)(3.8) = 2.66\text{m} = 2660\text{mm}$$

The slenderness ratio: $\lambda = \frac{L_e}{I} = \frac{L_e}{\sqrt{I/A}} = \frac{2660}{\sqrt{(10.71875x10^8)/(300x350)}}$

$$\Rightarrow \lambda = 26.327$$

$$\lambda \leq 50 - 25\left(\frac{M_1}{M_2}\right) = 50 - 25\left(-\frac{30}{45}\right) = 66.67 \rightarrow Ok!$$

Therefore, the column is short.

(b) For un braced column (sway structure)

$$\lambda = \sqrt{\frac{12A}{K_i L}} = \frac{L}{\sqrt{I/A}}\left(\sin ce, K_i = \frac{12I}{L^3}\right)$$

$$But\ A = (300 \times 400) + (300 \times 350) = 225000mm^2\ and$$

$$I = \frac{300 \times 400^3}{12} + \frac{300 \times 350^3}{12} = 26.7187 \times 10^8 mm^4$$

$$\sqrt{\frac{I}{A}} = \sqrt{\frac{26.7187 \times 10^8 mm^4}{225000mm^2}} = 108.97 \Rightarrow \lambda = \frac{3800}{108.97} = 34.87$$

$$\lambda \leq 25\ \ or\ \ \lambda \leq \frac{15}{\sqrt{\frac{525x10^3}{0.85x^{30}/_{1.5}x300x350}}} = 27.66 \rightarrow not\ ok!$$

Therefore, the column is long.

Q2). Determine the quantity of reinforcement use concrete material $C-30\ \&\ Steel\ S-300\ classI\ works.$

3.4 Classification of Columns on the Basis of Loading

a) Axially loaded columns

They are columns subjected to axial or concentric load without moments. They occur rarely.

When concentric axial load acts on a short column, its ultimate capacity may be obtained, recognizing the nonlinear response of both materials, from:

$$P_{ult} = f_{cd}(A_g - A_{st}) + A_{st} f_{yd}$$

Where A_g is gross cross-sectional area $(b \times h)$

A_{st} is total reinforcement area

$$f_{cd} = \frac{\alpha f_{ck}}{\gamma_c} \quad \& \quad f_{yd} = \frac{f_{yk}}{\gamma_s}$$

f_{ck} is characteristic compressive cylinder strength of concrete

α is coefficient, generally taken as 0.85

$\gamma_c \& \gamma_s$ is partial factors of safety for concrete and steel

When concentric axial load acts on a long column$\left(\frac{L_e}{b} \geq 12\right)$, its ultimate capacity may be obtained from:

$$\boldsymbol{P_{dul} = C_r P_{do} \quad where\ C_r = 1.25 - \frac{L_e}{48b}}$$

Short columns usually fail by crushing. Slender column is liable to fail by buckling. The end moments on a slender column cause it to deflect sideways and thus bring into play an additional moment. The additional moment causes a further lateral deflection and if the axial load exceeds a critical value, this deflection and the additional moment become self -propagating until the column buckles.

For Pin ended columns:

$$\boldsymbol{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

A column is classified as short if both L_{ex}/h and L_{ey}/b are: $\leq 15\ for\ a\ braced\ column$

$\leq 10\ for\ an\ unbraced\ column$

Example2. The $300 \times 400mm$ Column shown below is having four 16mm diameter bars. Calculate the ultimate capacity to resist compressive and tensile force if $(a) L = 5.5m$ $(b)\ L = 7m.$ Take, $f_{yd} = 460\ N/mm^2$ $f_{ck} = 35\ N/mm^2$

$E_c = 9.5(f_{ck} + 8)^{1/3}$ $E_s = 200GPa$

Solution: The Column is braced.

(a) For $L = 5.5m$

$L_e = 0.7L$ (for one end fixed the other pinned – monograph)

$L_e = 0.7L = 0.7 * 5.5 = 3.85m$

$$\frac{L_{ex}}{a} = \frac{3.85}{0.4} = 9.625 \leq 15 \rightarrow short.$$

$$\frac{L_{ey}}{b} = \frac{3.85}{0.3} = 12.83 \leq 15 \rightarrow short.$$

The Column with this length and restrain Conditions is a short column.

$$A_g = 300 \times 400 = 120000mm^2 \qquad (gross\ area)$$

$$A_s = \frac{8\pi(16)^2}{4} = 1608mm^2 \qquad (reinforcement\ area)$$

Design Compressive force:

$$N_{sd} = \frac{0.85 f_{ck}}{\gamma_c}\left(A_g - A_s\right) + \frac{f_y}{\gamma_s} A_s$$

Taking $\gamma_c = 1.5$

$\gamma_s = 1.15$

$$N_{sd} = \frac{(0.85)(35)}{1.5}(120000 - 1608) + \frac{(460)(1608).}{1.15}$$

$$= 2991308N = 2991.31KN$$

In tension, the design axial load is:

$$N_{sd} = -\frac{f_y}{r} A_s = -\frac{460}{1.15}(1608)N = -643.2kN$$

(b) For $L = 7m$

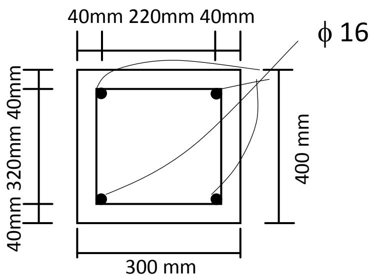
$L_e = (0.7)(7) = 4.9m$

$$\frac{L_{ex}}{n} = \frac{4.90}{0.4} = 12.25 \leq 15 \rightarrow short$$

$$\frac{L_{ey}}{b} = \frac{4.90}{0.3} = 16.33 > 15 \rightarrow Long.$$

Therefore, the column is slender.

$$N_{sd} = \frac{\pi^2 EI_e}{L_e^2} \qquad EI_e = 0.2E_cI_c + E_sI_s.$$



$$I_c = \frac{(400)(300)^3}{12} = 9x10^8 mm^4$$

$$I_s = \frac{(4)(\pi)(16)^2}{4}(144)^2 = 9.73x10^6 mm^4.$$

..

Take minimum reinforcement Cover $= 32mm$

$$\therefore EI_e = (0.2)(33)(9x10^8) + (200)(9.73x10^6).$$

$$= 59.4x10^8 + 19.46x10^8 = 78.86x10^8 kN.mm^2$$

$$\therefore N_{cr} = \frac{(\pi^2)(78.86x10^8)}{(4900)^2} = \underline{\underline{3241.6kN.}}$$

3 A column resting on independent footing supports of solid slab the super imposed factored load transferred from the slab is 1000KN. Design the column assuming a gross steel ratio of

a) 0.01

b) 0.02. Use concrete C-30, steel S-300 class I works. Assume column height 4m and $L_e = h$

I. Neglect buckling effect

II. Consider buckling effect

Solution:

- **Design consents:**

$$C-30, \quad f_{cd} = \frac{0.85f_{ck}}{\gamma_c} \quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S-300, \quad f_{yk} = 300\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165 MPa$$

i. Neglecting buckling effect

Assuming as short column

$$P_{ult} \leq A_g\left[f_{cd}\left(1-\rho_g\right) + \rho_g f_{yd}\right]$$

a. $\rho_g = 0.01\ (\,assuming\ square\ cross\ section)$

$$P_{ult} \leq A_g[13.6(1-0.01) + (0.01 * 260.87)]$$

$$1000KN \leq A_g[16.073]$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{16.073} = \ 62{,}217$$

$$S \geq 249.43$$

$$use\ 250x250$$

$$A_{st} = \rho_g A_g = 0.01 \times (250mm \times 250mm)$$

$$\Rightarrow A_{st} = 625\ mm^2 (4\Phi_{16} = 804\ mm^2)$$

- **Lateral Reinforcement:**

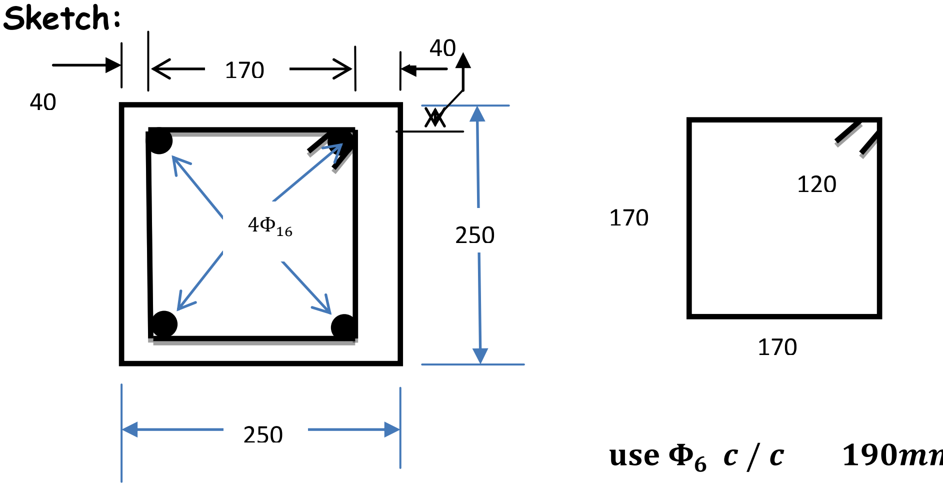
Ties or spiral:

$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{16}{4} = 4mm \end{cases} \qquad \Rightarrow \Phi = 6mm$$

$$\text{S} \leq \begin{cases} 300\ mm \\ b = 250\ mm \\ 12\Phi = 12x16mm = 192mm \end{cases} \qquad \Rightarrow S = 190mm$$

$$\text{use } \Phi_6 \quad c/\ c\ \ 190mm$$

- **Sketch:**



$$\textbf{use } \Phi_6 \;\; c\,/\,c \quad 190mm \quad L = 800mm$$

b. $\rho_g = 0.02$

$$P_{ult} \leq A_g[13.6(1-0.02)+(0.02*260.87)]$$

$$1000KN \leq A_g[18.545]$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{18.545} = 53{,}922$$

$$S \geq 232.21$$

$$use\ 240x240$$

$$A_{st} = \rho_g A_g = 0.02 \times (240mm \times 240mm)$$

$$\Rightarrow A_{st} = 1152\ mm^2 (4\Phi_{20} = 1257\ mm^2)$$

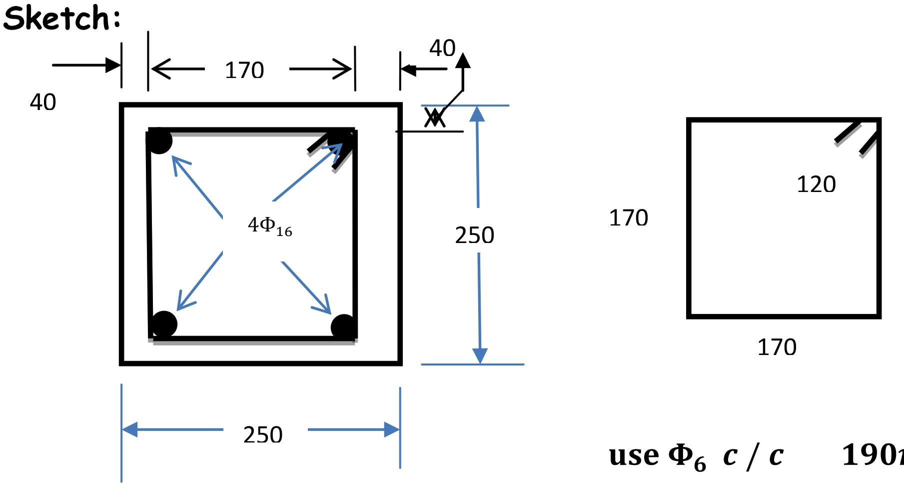
Ties or spiral:

$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{20}{4} = 5mm \end{cases} \qquad \Rightarrow \Phi = 5\ mm$$

$$S \leq \begin{cases} 300\ mm \\ b = 240\ mm \\ 12\Phi = 12x20mm = 240mm \end{cases} \qquad \Rightarrow S = 240mm$$

$$\text{use } \Phi_6 \;\; c/\,c \;\; 240mm$$

- **Sketch:**



$$\textbf{use } \Phi_6 \ c/c \quad 190mm \quad L = 800mm$$

Take minimum reinforcement cover =32mm

ii. Consider buckling effect

a. Try $250 \times 250\ mm$

$$C_r = 1.25 - \frac{L_e}{48b} = 1.25 - \frac{4000}{48 \times 250} = 0.917$$

$$P_u = C_r P_{ult}$$

$$P_{ult} \leq A_g\left[f_{cd}\left(1-\rho_g\right) + \rho_g f_{yd}\right]$$

$$\rho_g = 0.01\ (\,assuming\ square\ cross\ section)$$

$$P_{du} \leq C_r A_g[13.6(1-0.01) + (0.01 * 260.87)]$$

$$1000KN \leq C_r A_g[16.073]$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{16.073 * 0.917} = 67{,}847.48$$

$$S \geq 260.48mm > S_{used}$$

- **Revise**

Use $S = 260\ mm$

$$260x\ 260\ , C_r = 1.25 - \frac{L_e}{48b} = 1.25 - \frac{4000}{48 \times 260} = 0.929$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{16.073 * 0.929} = 66935.98$$

$$S \geq 258.72\ mm < S_{used}\ , \quad it\ is\ ok!$$

a) $\rho_g = 0.01$

$$A_{st} = \rho_g A_g = 0.01 \times (260mm \times 260mm)$$

$$\Rightarrow A_{st} = 676\ mm^2 (4\Phi_{16} = 804\ mm^2)$$

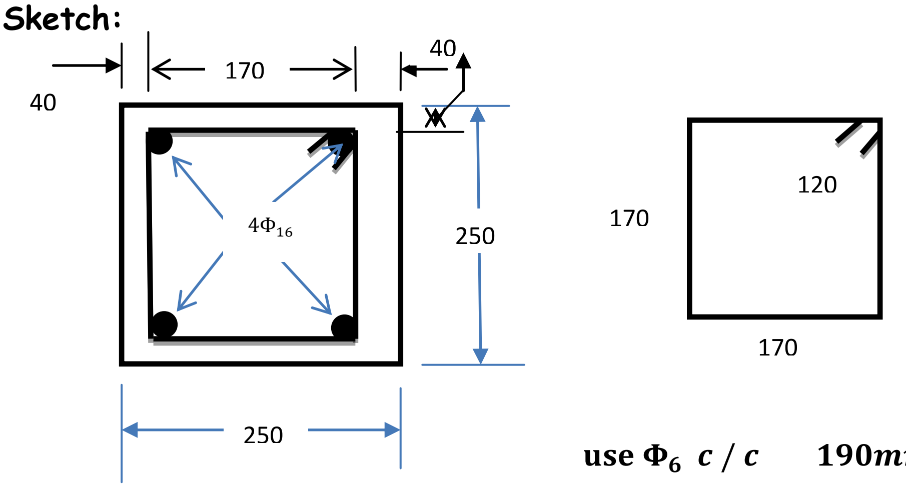
Ties or spiral:

$$\Phi \geq \begin{cases} 6mm \\ \dfrac{\Phi}{4} = \dfrac{16}{4} = 4mm \end{cases} \qquad \Rightarrow \Phi = 6\ mm$$

$$S \leq \begin{cases} 300\ mm \\ b = 260\ mm \\ 12\Phi = 12x16mm = 192mm \end{cases} \qquad \Rightarrow S = 190mm$$

$$\text{use } \Phi_6 \quad c/c \ \ 190mm$$

- **Sketch:**



$$\textbf{use } \boldsymbol{\Phi_6 \ \ c / c \quad 190mm \quad L = \ 800mm}$$

b) $\rho_g = 0.02$

$$Try\ 240 \times 240$$

$$C_r = 1.25 - \frac{L_e}{48b} = 1.25 - \frac{4000}{48 \times 240} = 0.903$$

$$P_{du} = C_r P_{ult}$$

$$P_{ult} \leq A_g\left[f_{cd}\left(1 - \rho_g\right) + \rho_g f_{yd}\right]$$

$$P_{du} \leq C_r A_g [13.6(1 - 0.01) + (0.01 * 260.87)]$$

$$1000KN \leq C_r A_g[16.073]$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{16.073 * 0.903} = 68{,}900.66$$

$$S \geq 262.52\, mm > S_{used} \qquad not\ ok!$$

- **Revise**

Use $S = 260\, mm$

$$Try\ with\ 260x\ 260\ , C_r = 1.25 - \frac{L_e}{48b} = 1.25 - \frac{4000}{48 \times 260} = 0.929$$

$$\rightarrow S^2 \geq \frac{1000 \times 10^3}{16.073 * 0.929} = 66{,}935.98$$

$$S \geq 258.72\, mm < S_{used}\ , \quad it\ is\ ok!$$

$$use\ 260\ x\ 260$$

$$A_{st} = \rho_g A_g = 0.02 \times (260mm \times 260mm) = 1352\, mm^2$$

$$A_g = 260 * 260 = 67600\, mm^2$$

$$\Rightarrow A_{st} = 1352\, mm^2 (4\Phi_{24} = 1809.55\, mm^2)$$

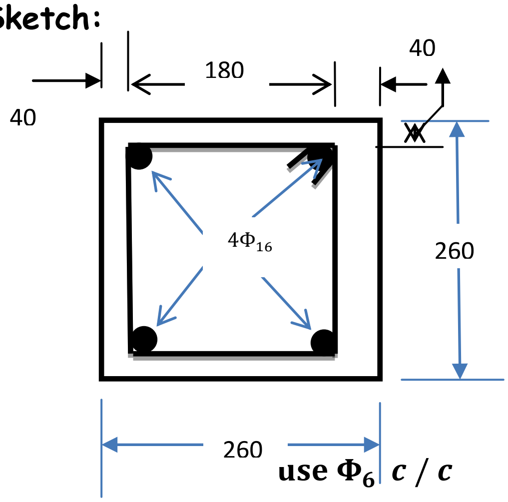
Ties or spiral:

$$\Phi \geq \begin{cases} 6mm \\ \dfrac{\Phi}{4} = \dfrac{24}{4} = 6mm \end{cases} \qquad \Rightarrow \Phi = 6\, mm$$

$$S \leq \begin{cases} 300\, mm \\ b = 260\, mm \\ 12\Phi = 12x24mm = 288mm \end{cases} \qquad \Rightarrow S = 260mm$$

$$\text{use } \Phi_6 \quad c/c \;\; 260mm$$

- **Sketch:**



use Φ_6 c/c $190mm$ $L = 840mm$

b) Column under uniaxial bending

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to the load not being centered on the column or may result from the column resisting a portion of the unbalanced moments at the end of the beams supported by columns.

When a member is subjected to combined axial compression P_d and moment M_d, it is more convenient to replace the axial load and the moment with an equivalent P_d applied at eccentricity e_d as shown below.

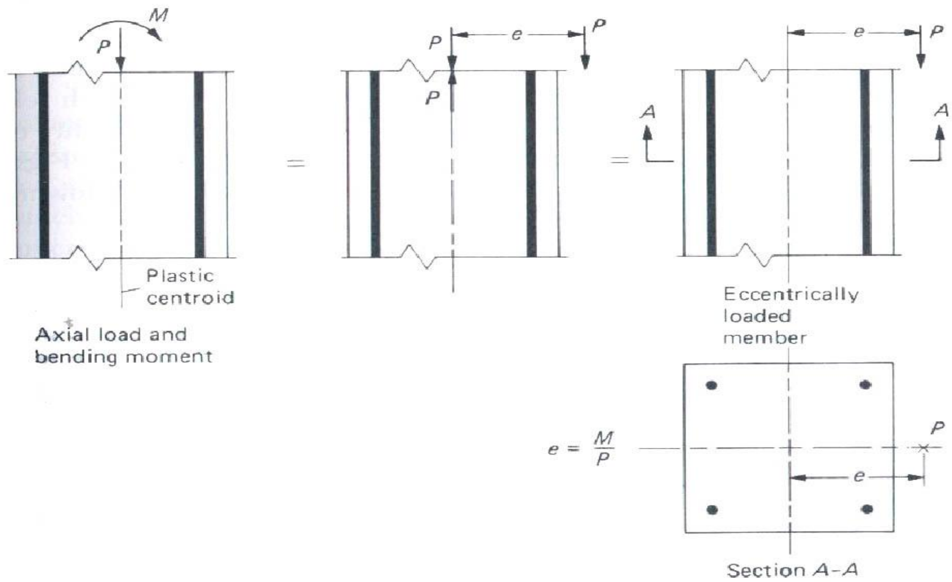


Fig. 3.11 Equivalent eccentricity of column load

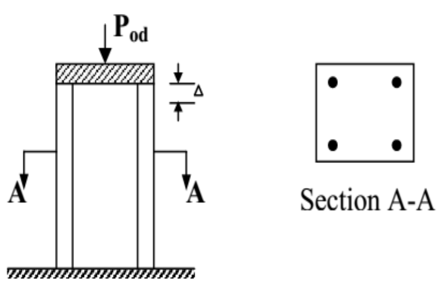
Design of Short Members for Axial Force and Uniaxial Bending

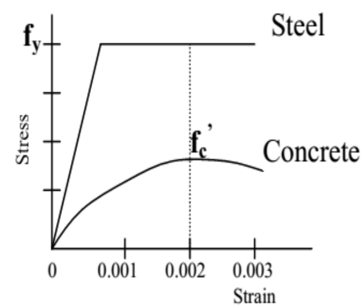
General:

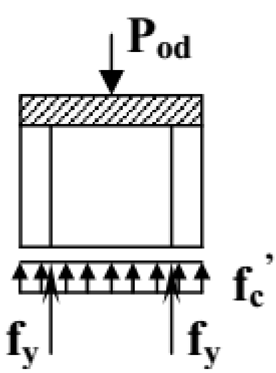
A column is subjected to uniaxial bending when the load applied to a column is eccentric about one axis only. The presence of this form of bending in axially loaded members can reduce the axial load capacity of the member. It is the combined effect of axial compression and bending at the ultimate limit state that tends to govern the design.

Design load for axially loaded columns (ideal columns)

- No moment considered.







$$P_{od} = P_s + P_c = \frac{A_{st}f_y}{\gamma_s} + \frac{(A_g - A_{st})0.85f_c'}{\gamma_c}$$

$$\uparrow P_s = \frac{A_{st}f_y}{\gamma_s}$$

$$\uparrow P_c = \frac{(A_g - A_{st})0.85f_c'}{\gamma_c}$$

In practice column loads will have eccentricities at least due to imperfect constructions.

3.5. Design of columns, EBSC 2

I. General

The internal forces and moments may generally be determined by elastic global analysis using either first order theory or second order theory.

a) First-order theory, using the initial geometry of the structure, may be used in the following cases

- Non-sway frames
- Braced frames
- Design methods which make indirect allowances for second-order effects.

b) Second-order theory, taking into account the influence of the deformation of the structure, may be used in all cases.

II. Design of Non sway Frames

Individual non-sway compression members shall be considered to be isolated elements and be designed accordingly.

Design of Isolated Columns

For buildings, a design method may be used which assumes the compression members to be isolated. The additional eccentricity induced in the column by its deflection is then calculated as a function of slenderness ratio and curvature at the critical section

Total eccentricity

1. The total eccentricity to be used for the design of columns of constant cross-section at the critical section is given by:

$$e_{tot} = e_e + e_a + e_2$$

Where: e_e, is equivalent constant first-order eccentricity of the design axial load

e_a is the additional eccentricity allowance for imperfections. For isolated columns:

$$e_a = \frac{L_e}{300} \geq 20\,\text{mm}$$

e_2 is the second-order eccentricity

First order equivalent eccentricity

1. For first-order eccentricity e_0 is equal at both ends of a column

$$e_e = e_0$$

2. For first-order moments varying linearly along the length, the equivalent eccentricity is the higher of the following two values:

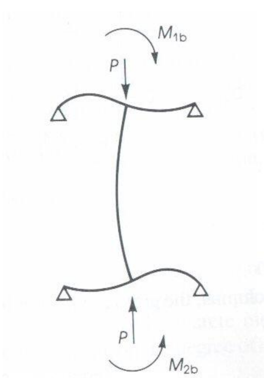
$$e_e = 0.6e_{02} + 0.4e_{01}$$
$$e_e = 0.4e_{02}$$

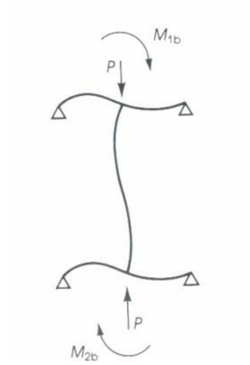
Where:

- e_{01} and e_{02} are the first-order eccentricities at the ends, e_{02} being positive and greater in magnitude than e_{01}.
- e_{01} is positive if the column bents in single curvature and negative if the column bends in double curvature.

3. For different eccentrics at the ends, (2) above, the critical end section shall be checked for first order moments:

$$e_{tot} = e_{02} + e_a$$





Second order eccentricity

1. The second-order eccentricity e_2 of an isolated column may be obtained as

$$e_2 = \frac{k_1 L_e^2}{10}\left(\frac{1}{r}\right)$$

Where L_e is the effective buckling length of the column

$$k_1 = \frac{\lambda}{20} - 0.75 \qquad For\ 15 \le \lambda \le 35$$

$$k_1 = 1.0 \qquad For \lambda > 35$$

$1/r$ is the curvature at the critical section.

2. The curvature is approximated by:

$$\frac{1}{r} = k_2\left(\frac{5}{d}\right)10^{-3}$$

Where:

- d is the effective column dimension in the plane of buckling
- $k_2 = M_d/M_b$
- M_d is the design moment at the critical section including second-order effects
- M_b is the balanced moment capacity of the column.

3. The appropriate value of k_2 may be found iteratively taking an initial value corresponding to first-order actions.

III. Design of Sway Frames

The second order effects in the sway mode can be accounted using either of the following two methods:

a) Second-order elastic global analysis: When this analysis is used, the resulting forces and moment may directly be used for member design.

b) Amplified Sway Moments Method: In this method, the sway moments found by a first-order analysis shall be increased by multiplying them by the moment magnification factor:

$$\delta = \frac{1}{1 - \frac{N_{sd}}{N_{cr}}}$$

Where N_{sd} is the design value of the total vertical load

N_{cr} is its critical value for failure in a sway mode.

The amplified sway moments method shall not be used when the critical load ratio $\frac{N_{sd}}{N_{cr}} > 0.25$

Sway moments are those associated with the horizontal translation of the top of story relative to the bottom of that story. They arise from horizontal loading and may also arise from vertical loading if either the structure or the loading is asymmetrical.

As an alternative to determining $\frac{N_{sd}}{N_{cr}}$ direct, the following approximation may be used in beam and-column type frames $\frac{N_{sd}}{N_{cr}} = \frac{N\delta}{HL}$ (see section 3.2)

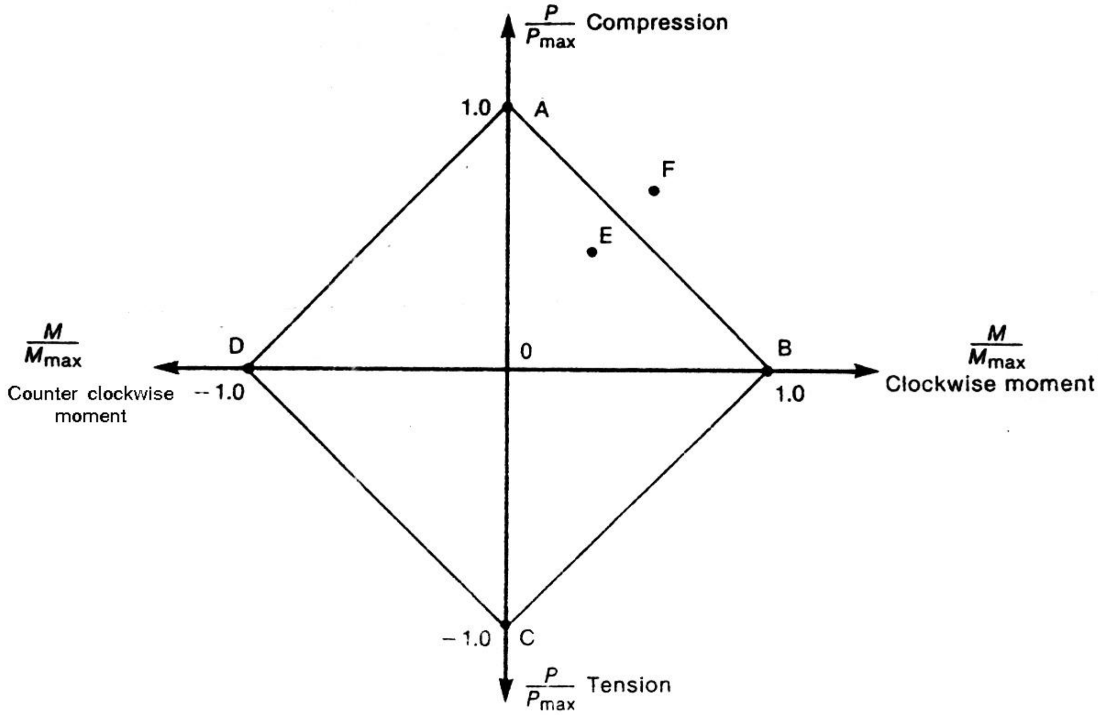
Where δ, , H and N are as defined before.

In the presence of torsional eccentricity in any floor of a structure, unless more accurate methods are used, the sway moments due to torsion should be increased by multiplying them by the larger moment magnification factor δ_s, obtained for the two orthogonal directions of the lateral loads acting on the structure.

3.6. Interaction diagram

The presence of bending in axially loaded members can reduce the axial load capacity of the member

To illustrate conceptually the interaction between moment and axial load in a column, an idealized homogenous and elastic column with a compressive strength, f_{cu}, equal to its tensile strength f_{tu}, will be considered. For such a column failure would occurs in a compression when the maximum stresses reached f_{cu} as given by:



$$\mathbf{f_{cu}} = \frac{\mathbf{P}}{\mathbf{A}} + \frac{\mathbf{MY}}{\mathbf{I}}$$

Dividing both sides by f_{cu} gives:

$$\frac{P}{f_{cu}A} + \frac{MY}{f_{cu}I} = 1$$

The maximum axial load the column could support is obtained when $M = 0$, and is $P_{max} = f_{cu}A$.

Similarly the maximum moment that can be supported occurs when $P = 0$ and is $M_{max} = f_{cu}I/y$.

Substituting P_{max} and M_{max} gives:

$$1 = \frac{P}{P_{max}} + \frac{M}{M_{max}} \qquad (*)$$

This is known as interaction equations because it shows the interaction of or relationship between P and M at failure. It is plotted as line AB (see Fig.). A similar equation for a tensile load, P, governed by $\mathrm{f_{tu}}$, gives line BC in the figure. The plot is referred to as an interaction diagram.

Points on the lines represent combination of P and M corresponding to the resistance of the section. A point inside the diagram such as E represents a combination of $\boldsymbol{P}$ ***and*** $\boldsymbol{M}$ that will not cause failure. Load combinations falling on the line or outside the line, such as point F will equal or exceed the resistance of the section and hence will cause failure.

Interaction Diagrams for Reinforced concrete Columns

Since reinforced concrete is not elastic and has a tensile strength that is lower than its compressive strength, the general shape of the diagram resembles Fig. 3.12

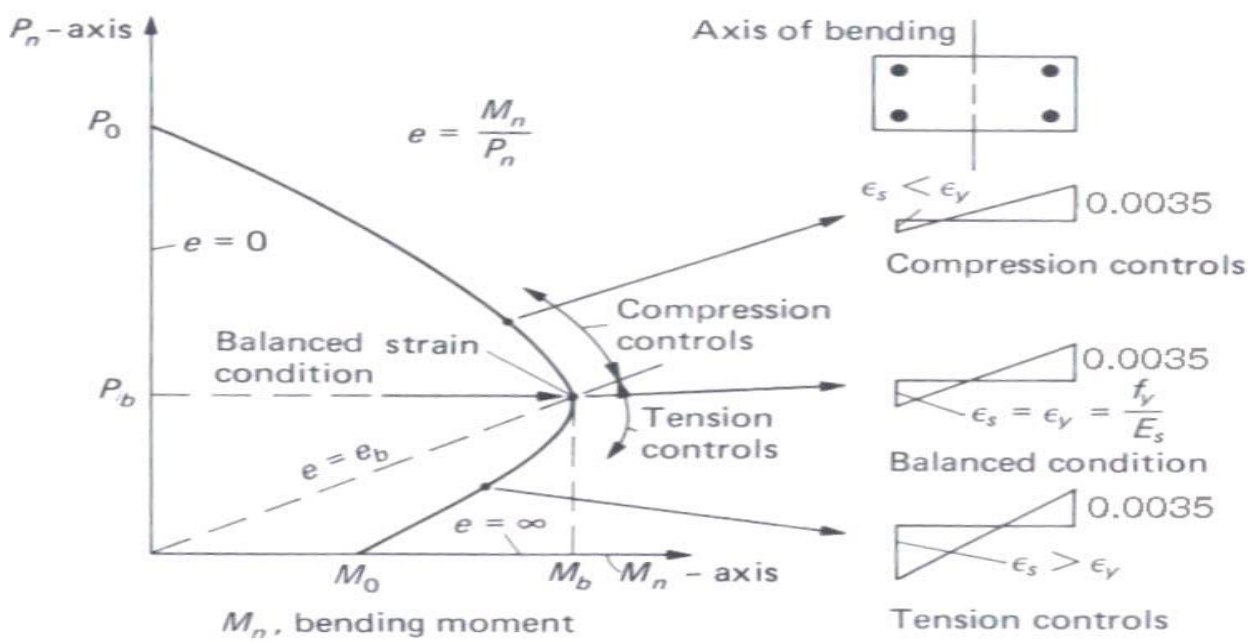


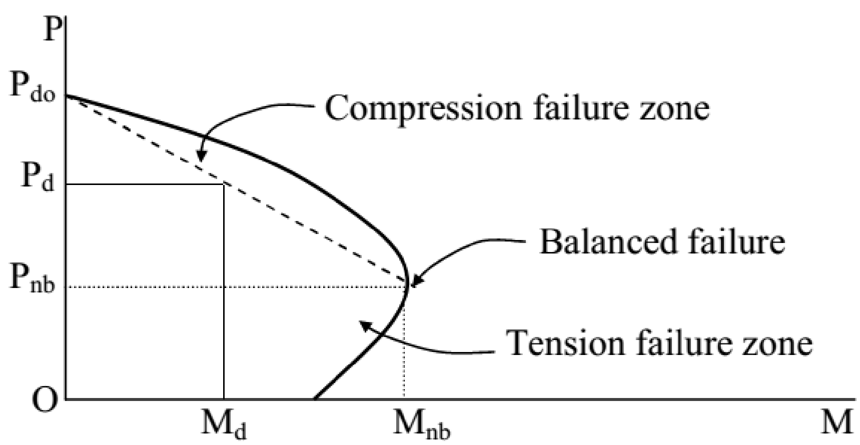
Fig. 3.12 Interaction diagram for column in combined bending and axial load

Balanced condition: For a given cross section the design axial force P_b acts at one specific eccentricity e_b to cause failure by simultaneous yielding of tension steel and crushing of concrete (see Fig. 3.12)

Tension failure controls: For a very large eccentricity of the axial force P_n, the failure is triggered by yielding of the tension steel. The horizontal axis corresponds to an infinite value of e, i.e. pure bending at moment capacity M_o (see Fig. 3.12)

Compression failure controls: For a very small eccentricity of the axial force P_n, the failure is governed by concrete compression. The vertical axis corresponds to $e = 0$ and P_o is the capacity of the column if concentrically loaded (see Fig. 3.12)

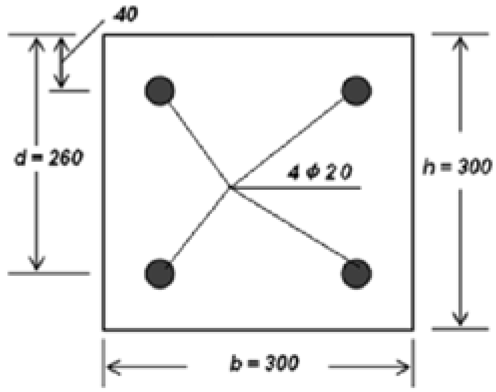
- It is a plot of a column axial load capacity against the moment it sustains



- Any loading within the curve is a possible safe loading combination.
- Any combination of loading outside the curve represents a failure combination.
- Any radial line from point O represents a constant ratio of moment to load⇒Constant eccentricity.
- The full line curve in compression failure range can be conservatively replaced by the dashed line as shown. Knowing the coordinates(O, P_{do}) & (M_{nb}, P_{nb}) , the design capacity $P_d$for a known moment M_d, $[M_d = e_d P_d]$ can be obtained using the straight line equation:

Example 4:

For the column section shown in the figure, compute the axial force and bending moment pairs of the significant points on the interaction Diagram and sketch the diagram. The characteristic concrete cube strength is $25\,KN/mm^2$ and characteristic yield strength of the steel is $300\,KN/mm^2$



Solution:

$$f_{cu} = 25\,\mathrm{N/mm^2} \qquad \Rightarrow f_{ck} = \frac{f_{cu}}{1.25} = \frac{25}{1.25} = 20\mathrm{MPa}$$

$$f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = \frac{0.85 \times (20)}{1.5} = 11.33\mathrm{MPa}$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87MPa$$

b) Pure bending

$$P = f_{cd}(0.8x)b + f_{sc}A_s' - f_s A_s = 0$$

$f_s = f_{yd}$ *and the stress in the compression steel ca be expresses interms of still unknown N.A*

$$f_{sc} = f_s' = E_s \varepsilon_{cu} \frac{(x - d')}{x}$$

Where: $\varepsilon_{cu} = 0.0035 = 3.5mm$ & $E_s = 200GPa$

$$\Rightarrow f_{sc} = f_s' = E_s \varepsilon_{cu} \frac{(x - d')}{x} = 3.5 * 200 \left(\frac{(x - 40)}{x}\right) = \frac{700(x - 40)}{x}$$

$$\Rightarrow f_{cd}(0.8x)b + f_{sc}A_s' - f_s A_s = 0$$

Where:

$$A_s' = 2\left(\frac{\pi * D^2}{4}\right) = \left(\frac{\pi * 20^2}{2}\right) = 628mm^2 \ \&$$

$$A_s = 2\left(\frac{\pi * D^2}{4}\right) = \left(\frac{\pi * 20^2}{2}\right) = 628mm^2$$

$$\Rightarrow [11.33 * (0.8x) * 300] + \left(\frac{700(x-40)}{x} * 628mm^2\right) - (628mm^2 * 260.87) = 0$$

$$\Rightarrow 2719.2x + \frac{439600(x-40)}{x} - 163826.36 = 0$$

$$\Rightarrow 2719.2x^2 + 275773.64x - 17584000 = 0$$

$$\Rightarrow x^2 + 101.417x - 6466.608 = 0$$

Solving the above equation (as a quadratic equation or using trial –and –error

$$\Rightarrow x = 44.4mm$$

$$\Rightarrow f_{sc} = f_s' = \frac{700(x-40)}{x} = \frac{700(44.4-40)}{44.4} = 69.4MPa$$

The ultimate moment M_o:

$$M_o = C_c(0.5h - 0.4x) + C_s(0.5h - d') + T(d - 0.5h)$$

Where:

$$C_c = f_{cd}(0.8x)b = 11.33 * (0.8 * 44.4) * 300 = 120732.48N = 120.7325KN$$

$$C_s = f_{sc}A_s' = 69.4MPa * 628mm^2 = 43583.2N = 43.583KN$$

$$T = A_s f_{yd} = 628mm^2 * 260.87MPa = 163{,}826.36N = 163.826KN$$

$$\Rightarrow M_o = 120.733(150 - 0.4 * 44.4) + 43.583 * (150 - 40) + 163.826 * (260 - 150)$$
$$= (120.723KN * 132.24mm) + [43.583KN * 110mm] + [163.826KN * 110mm])$$
$$= 38{,}780.66KNmm = 38.78\ KNm$$

c) Balanced Failure:

$$x_{bal} = \frac{(\varepsilon_{cu} * d)}{\varepsilon_{cu} + \varepsilon_{yd}}, \quad where \quad \varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{260.87}{200000} = 0.001304 \cong 0.0013$$

$$= \frac{0.0035 * 260}{(0.0035 + 0.0013)} = 189.6mm$$

$$\varepsilon_s' = \frac{\varepsilon_{cu}(x-d')}{x} = \frac{0.0035(189.6-40)}{189.6} = 0.00276 > \varepsilon_{yd}$$

$$P_b = f_{cd}(0.8x)b + f_{yd}A'_s - f_{yd}A_s$$

$$= 11.33 * 0.8 * 189.6 * 300N = 515.56KN$$

$$\begin{aligned} M_b &= C_c(0.5h-0.4x) + C_s(0.5h-d') + T(d-0.5h) \\ &= [515.56KN[(150-(0.4*189.6)] + 2*628*260.87*(150-40)] \\ &= [38233.9296KNmm] + (36041.7992KNmm) \\ &= 74.276KNm \end{aligned}$$

d) Pure axial force:

$$\begin{aligned} P_o &= f_{cd}(A_c) + f_{yd}A_{st} \\ &= 11.33 * \left(300^2mm^2 - 4*\left(\frac{\pi*20^2}{4}\right)\right) + \left(260.87*4*\left(\frac{\pi*20^2}{4}\right)\right) \\ &= 1333.28KN \end{aligned}$$

Other positions of N.A:

e) $\mathbf{x = 260mm(Tension\ strain = 0)}$:

$$\varepsilon_s = 0$$

$$\varepsilon'_s = \varepsilon_{cu} * \frac{x-d}{x} = 0.0035 * \left(\frac{(260-40)}{260}\right) = 0.00296 \cong 0.003 > \varepsilon_{yd}$$

$$\begin{aligned} P &= f_{cd}(0.8x)b + f_{yd}A_{sc} \\ &= [11.33*(0.8*260)*300] + (260.87*628mm^2) \\ &= 870.82KN \end{aligned}$$

$$\begin{aligned} M_b &= C_c(0.5h-0.4x) + C_s(0.5h-d') \\ &= [870.82[(150-(0.4*260)] + \ 628mm^2*260.87*(150-40)] \\ &= [40057.72KNmm] + (18020.8996KNmm) \\ &= 58.079KNm \end{aligned}$$

f) Yielding of compression steel:

$$\varepsilon'_s = \varepsilon_{cu} * \frac{(x-d')}{x} = \varepsilon_{yd}$$

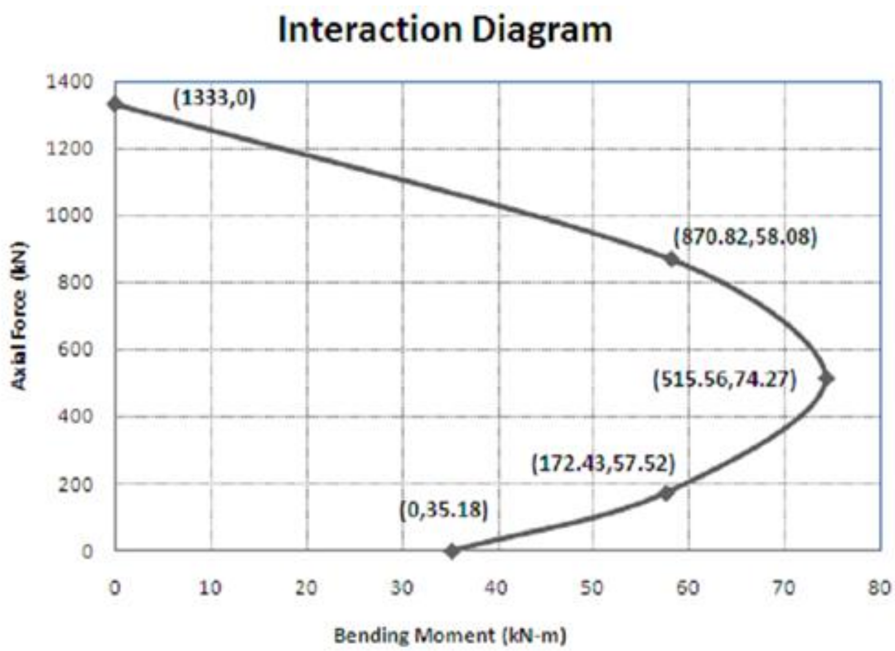
$$\Rightarrow x = \frac{(\varepsilon_{cu}*d')}{\varepsilon_{cu}+\varepsilon_{yd}}, \qquad where \quad \varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{260.87}{200000} = 0.001304 \cong 0.0013$$

$$= \frac{0.0035 * 40}{(0.0035 + 0.0013)} = 63.76mm$$

$$P = f_{cd}(0.8x)b + f_{yd}A'_s - f_{yd}A_s$$
$$= [11.33 * (0.8 * 63.76) * 300] = 173.38KN$$

$$M_b = C_c(0.5h - 0.4x) + C_s(0.5h - d') + T(d - 0.5h)$$
$$= [173.38KN[(150 - (0.4 * 63.76)] + 2 * 628 * 260.87 * (150 - 40)]$$
$$= [21585.116KNmm] + (36041.7992KNmm)$$
$$= 57.63KNm$$

The interaction diagram is shown below:



3.7. Design of Columns for uniaxial Bending

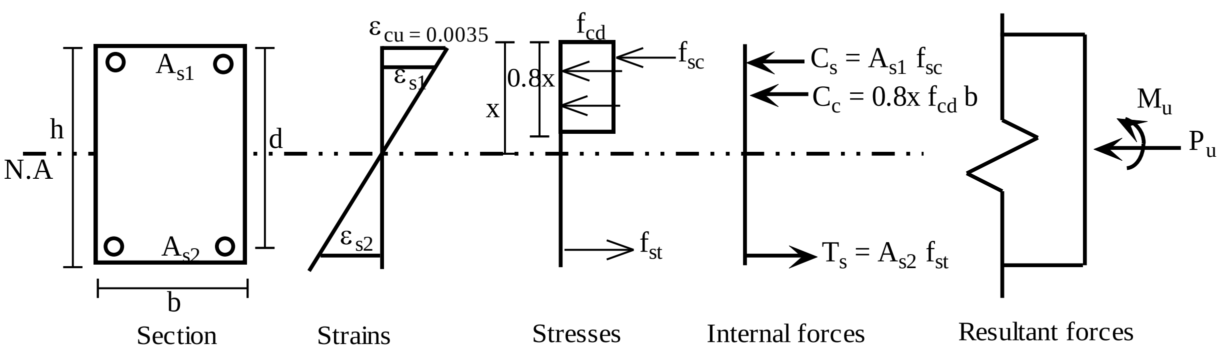
Given P_d and e_d , the design requires the following procedure.

- A trial cross section and steel area ρ_g are selected.
- The section in question is investigated which load combination it can sustain. More suitably, for a fixed value of e_d, determine P_{dn} (its capacity) such that.
  - ✓ If $P_{dn} \geq P_d$, safe but is it economical
  - ✓ If $P_{dn} < P_d$, Unsafe, choose new cross section and /or $\rho_g = \frac{A_{st}}{A_c}$

  Where:

$$P_{dn} = f_{cd}bh\left[\frac{0.8\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_s} - \rho\right] \qquad \rho = \frac{A_s}{bh} = \frac{A'_s}{bh}$$

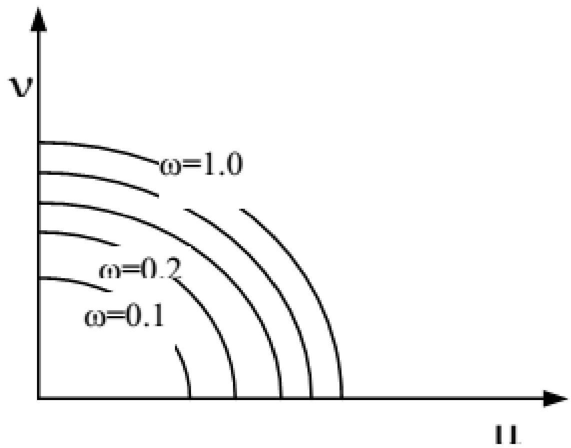
- Thus, the trial shall be repeated until the value of $P_{dn}$is close enough and $P_{dn} \geq P_d$
- Interaction diagrams for columns are generally computed by assuming a series of strain distributions, each corresponding to a particular point on the interaction diagram, and computing the corresponding values of P and M (strain compatibility analysis).
- The calculation process can be illustrated as follow for one particular strain distribution.



$$\Sigma F_h = 0 \quad \Rightarrow \quad P_u = C_c + C_s + T_s$$

$$M_u = \Sigma M_{\text{at the geometric centroid of the section}}$$

- On the other hand, interaction charts are prepared using non dimensional parameters such that



$\boldsymbol{\nu\ V_s\ \mu\ plotted}$

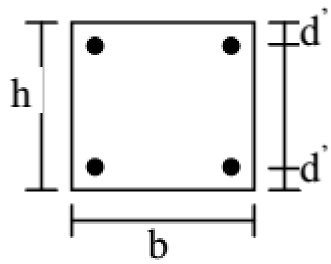
Normal force ratio,

$$\upsilon = \frac{N}{f_{cd}bh}$$

Moment ratios:

$$\mu = \frac{M_h}{f_{cd}bh^2}$$

Where $N = P_d$, $M_h = M_d$



h is in the direction of the bending moment.

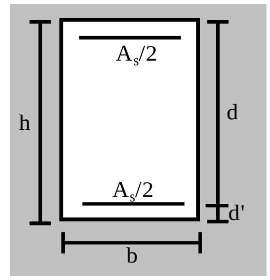
- Families of curves are drawn for fixed ratio [$ranges\ 0.5\ to\ 0.25$] each curve Representing a particular mechanical steel ratio.

$$\omega = \frac{A_{st}f_{yd}}{bhf_{cd}}$$

- The cover to reinforcement should not be too large [problem of spalling – concrete cover falling off] & also not too small to prevent corrosion /fire. Usually for column, cover$\geq 25mm$.

In the actual design, interaction charts prepared for uniaxial bending can be used. The procedure involves:

- Given P_d and e_d such that $M_d = P_d e_d$
  - Assume a cross section, d' and evaluate $\frac{d'}{h}$ to choose appropriate chart
  - Compute:
    - Normal force ratio: $\upsilon = \frac{N_u}{f_{cd}bh}$
    - Moment ratios:$\mu = \frac{M_u}{f_{cd}bh^2}$



- Select suitable chart which satisfy $\frac{d'}{h}$ ratio:
- Enter the chart and pick ω (the mechanical steel ratio), if the coordinate (υ, μ) lies within the families of curves. If the coordinate (υ, μ) lies outside the chart, the cross section is small and a new trail need to be made.
- Compute $A_{s,tot} = \frac{\omega A_c f_{cd}}{f_{yd}}$
- Check $A_{s,tota}$ satisfies the maximum and minimum provisions
- Determine the distribution of bars in accordance with the charts requirement

Example 2.7.2

Design a column to sustain a design axial load of $1100KN$ and design bending moment of $160KNm$,which includes all other effects, assume concrete $C-30$, steel $S-400$ class I work. Approximate $b = 0.6h$.

Solution

- **Design consents:**

$$C-30, \quad f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} \quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S - 400, \quad f_{yk} = 400\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{400}{1.15} = 347.83\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165MPa$$

$$Assume:\ bxh = 270x450, \qquad d' = 25 + 12 + 8 = 45mm$$

$$\rightarrow \frac{d'}{h} = \frac{45}{450} = 0.1 \quad [\ chart\ number\ 2)$$

$$\mu_{sd} = \frac{M_{sd}}{A_c f_{cd} h} = \frac{160 \times 10^6}{(270 \times 450) \times 13.6x450mm} = 0.22$$

$$\nu_{sd} = \frac{N_{sd}}{A_c f_{cd}} = \frac{1100 \times 10^3}{(270 \times 450) \times 13.6} = 0.67$$

Using uniaxial chart number 2,

$$\omega = 0.35$$

- **Determine Reinforcement:**

$$A_s = \frac{\omega A_c f_{cd}}{f_{cd}} = \frac{0.35 \times (270 \times 450) \times 13.6}{347.83} = 1663\ mm^2$$

$$\Rightarrow A_s = 1663\ mm^2 (use\ 4\Phi_{24} = 1809mm^2)$$

- **Check :$\boldsymbol{A_{s,min} \le A_{s,proved} \le A_{s,max}}$**
- **The minimum reinforcement:**

$$A_{s,min} = 0.008A_c = [0.008 * (270mm * 450m)] = 972mm^2$$
$$A_{s,max} = 0.08A_c = [0.08 * (270mm * 450m)] = 9720mm^2$$
$$A_{s,min} \le A_{s,proved} \le A_{s,\max}$$
$$\rightarrow 972mm^2 < 1809\ mm^2 \le 9720mm^2$$

Therefore, it is ok!

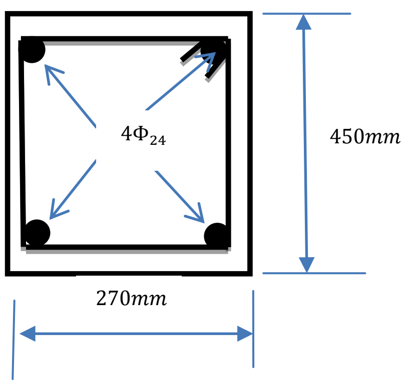
- **Lateral Reinforcement:**

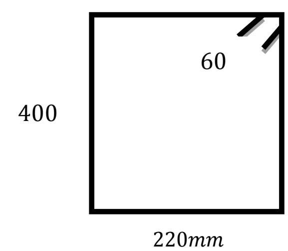
$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{24}{4} = 6mm \\ 8mm \end{cases} \qquad \Rightarrow \Phi = 8\,m$$

$$S \leq \begin{cases} 300\ mm \\ b = 270\ mm \\ 12\Phi = 12x24mm = 288mm \end{cases} \qquad \Rightarrow S = 270mm$$

$$\text{use } \Phi_8 \ \ c/\,c \ \ 270mm$$

- **Sketch:**





use Φ_8 $c\,/\,c$ $270mm$ $L = 1360mm$

Example 3.7.3

Design of slender braced columns subjected to uniaxial bending:

Given:

Action effect- factored axial loads $is\ 1650\ KN$

Factored first order equivalent constant moment$is\ 130\ KN\ m$

Geometry: $L = 7m\ \ \&L_e = 0.7L$

Materials: $C - 30\ \ ,S - 460\ \ \ class\ I\ works$

Required: Quantity of reinforcement

Solution:

- Assume:

- $Column\ size\ ,b/_h = 400/_{400\ mm}$
- $with\ cover\ is\ 20mm$
- $\Phi_{long} = 20mm \quad and \quad \Phi_{lat} = 10mm$
- $\mathrm{d}' = 20 + 10 + \frac{20}{2} = 40mm$
- $d'/_h = \frac{40}{400} = 0.1$

$$\Rightarrow d = h - d' = 400 - 40 = 360mm$$

- **Design consents:**

$$C-30, \quad f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} \quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S-460, \quad f_{yk} = 460\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{460}{1.15} = 400\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165MPa$$

- **Eccentricity:**

$$\boldsymbol{e_{tatal} = e_e + e_a + e_2}$$

- $\boldsymbol{Equivalent\ first\ order\ eccentricity\ of\ the\ axial\ load}$:

$$e_e = \frac{M_{sd}}{P_{sd}} = \frac{130 \times 10^3}{1650} = 78.8mm$$

- $\boldsymbol{Additional\ \ eccentricity}$:

$$e_a \geq \begin{cases} \frac{L_e}{300} = \frac{0.7 * 7000}{300} = 16.33 \\ 20 \end{cases}$$

$$\rightarrow e_a = 20\ mm$$

- **Check for second order effect:**

$$e_2 = \frac{k_1 L_e^2}{10}\left(\frac{1}{r}\right)$$

$$\lambda = \frac{L_e}{\sqrt{\frac{I}{A}}} = \frac{0.7 * 7000}{\sqrt{\frac{bD^3}{12bD^2}}} = \frac{0.7 * 7000}{\sqrt{\frac{(400)^2}{12}}} = 42.4$$

$$\lambda = 42.4 > 50 - 25\frac{M_1}{M_2} = 50 - 25\left(\frac{130}{130}\right) = 25$$

Therefore, second order effect should be considered.

- Critical value of k_2 corresponding to first order effect including additional eccentricity ,e_a

$$M_{sd} = P(e_e + e_a) = 1650KN\left(\frac{78.8}{1000} + \frac{20}{1000}\right)m = 163\ KN\ m$$

$$\nu_{sd} = \frac{N_{sd}}{A_c f_{cd}} = \frac{1650 \times 10^3}{(400 \times 400) \times 13.6} = 0.76$$

$$\mu_{sd} = \frac{M_{sd}}{A_c f_{cd} h} = \frac{163 \times 10^6}{(400 \times 400) \times 13.6x400mm} = 0.187$$

Using uniaxial chart Number 2,$\left({}^{d'}/_{h} = \frac{40}{400} = 0.1\right)$

$$\mu_b = 0.32\ ,$$

$$\mu_{bal} = 0.25$$

$$k_2 = \frac{\mu_{sd}}{\mu_{bal}} = \frac{0.187}{0.25} = 0.75$$

$$\left(\frac{1}{r}\right) = k_1\left(\frac{5}{d}\right)10^{-3} = 0.75 \times \left(\frac{5}{360}\right) * 10^{-3} = 10.42x10^{-6}mm^{-1}$$

$$k_1 = 1\ \ for\ \lambda > 35$$

$$\Rightarrow e_2 = \frac{k_1 L_e^2}{10}\left(\frac{1}{r}\right) = \frac{1 * (4900)^2}{10}(10.42x10^{-6}) = 25$$

$$\Rightarrow e_{Total} = e_e + e_a + e_2 = 78.8mm + 20mm + 25mm = 123.8\ mm$$

$$M_{sd} = Pe_{Total} = 1650\ KN\left(\frac{123.8}{1000}\right) = 204.3KN\ m$$

$$\mu_{sd} = \frac{M_{sd}}{A_c f_{cd} h} = \frac{204.3 \times 10^6}{(400 \times 400) \times 13.6x400mm} = 0.23$$

$$\nu_{sd} = 0.76$$

$$\mu_{bal} = 0.45$$

$$\Rightarrow k_2 = \frac{\mu_{sd}}{\mu_{bal}} \quad , \quad \mu_{bal} = 0.30$$

$$k_2 = \frac{\mu_{sd}}{\mu_{bal}} = \frac{0.76}{0.3} = 0.77$$

$$\left(\frac{1}{r}\right) = k_2 \left(\frac{5}{d}\right) 10^{-3} = 0.77 \times \left(\frac{5}{360}\right) * 10^{-3} = 10.69x10^{-6}mm^{-1}$$

$$e_2 = \frac{k_1 L_e^2}{10} \left(\frac{1}{r}\right) = \frac{1 * (4900)^2}{10} (10.69x10^{-6}) = 25.7mm$$

$$\Rightarrow e_2 = 26mm$$

$$\Rightarrow e_{Total} = e_e + e_a + e_2 = 78.8mm + 20mm + 26mm = 124.8\ mm$$

$$M_{sd} = Pe_{Total} = 1650\ KN \left(\frac{124.8}{1000}\right) = 205.92KN\ m$$

$$\mu_{sd} = \frac{M_{sd}}{A_c f_{cd} h} = \frac{205.92 \times 10^6}{(400 \times 400) \times 13.6x400mm} = 0.236$$

$$\omega = 0.45, hence\ iteration\ can\ be\ stopped$$

- **Determine Reinforcement:**

$$A_s = \frac{\omega A_c f_{cd}}{f_{cd}} = \frac{0.45 \times (400 \times 400) \times 13.6}{400} = 2448mm^2$$

$$\Rightarrow A_s = 2448mm^2 (use\ 8\Phi_{20} = 2513mm^2)$$

- **The minimum reinforcement:**

$$A_{s,min} = 0.008A_c = [0.008 * (400mm * 400m)] = 1280mm^2$$
$$A_{s,max} = 0.08A_c = [0.08 * (400mm * 400m)] = 12{,}800mm^2$$
$$A_{s,min} \leq A_{s,proved} \leq A_{s,\max}$$
$$\rightarrow 1280mm^2 < 2513mm^2 < 12{,}800mm^2$$

Hence, A_s is which in the required range.

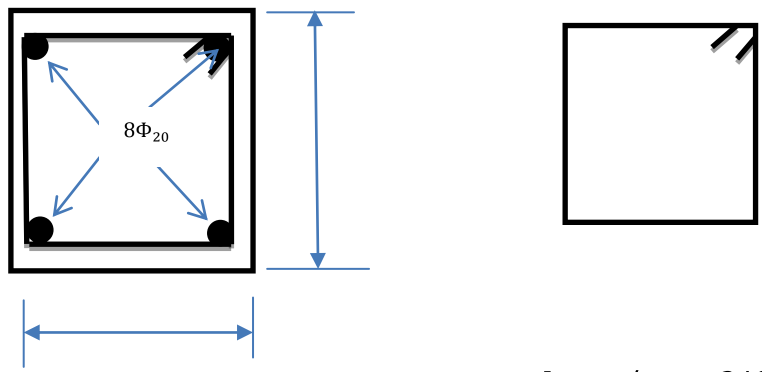
- **Lateral Reinforcement:**

$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{20}{4} = 5mm \\ 10mm \end{cases} \qquad \Rightarrow \Phi = 10\ mm$$

$$S \leq \begin{cases} 300\ mm \\ b = 400\ mm \\ 12\Phi = 12x20mm = 240mm \end{cases} \qquad \Rightarrow S = 240mm$$

use Φ_{10} c/c $240mm$

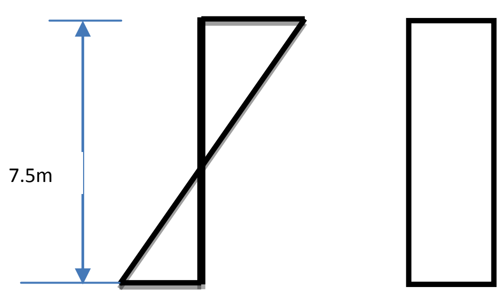
- **Sketch:**



use Φ_{10} c/c $240mm$ $L = 1560mm$

Example 3.7.4

A uniaxial column is to be constructed from materials C-30, S-400 Class I work. If the diagrams for first order end moments and axial force are as shown. Determine the area of reinforcement, assume on sway frame system use b/h =300/400, $L_e = 0.75L$



$P = 1280KN$
$M = -82KNM$

Solution:

$$Assume\ d' = 40mm \rightarrow \frac{d'}{h} = \frac{40}{400} = 0.1$$

$$\Rightarrow d = 400 - 40 = 360mm$$

$$e_{total} = e_e + e_a + e_2$$

$$e_e \geq \begin{cases} 0.06e_{02} + 0.4e_{01} \\ 0.4e_{02} \end{cases}$$

$$e_{02} = \frac{155}{1280} = 121.1mm$$

$$e_{01} = \frac{-82}{1280} = -64.1mm$$

$$e_e \geq \begin{cases} (0.6 * 121.1) + (0.4 * -64.1) = 47.02mm \\ 0.4e_{02} = 0.4 * 121.1 = 48.44mm \end{cases}$$

$$\Rightarrow e_e = 48.4mm$$

$$e_a \geq \begin{cases} \frac{L_e}{300} = \frac{0.75 * 7500}{300} = 18.75mm \\ 20mm \end{cases}$$

$$\Rightarrow e_a = 20\ mm$$

- **Check for second order effect**

$$\lambda = \frac{L_e}{\sqrt{\frac{I}{A}}} = \frac{0.75*7500}{\sqrt{\frac{bD^3}{12bD^2}}} = \frac{0.75*7500}{\sqrt{\frac{(400)^2}{12}}} = 48.71$$,

$$\lambda = 48.71 > 50 - 25\frac{M_1}{M_2} = 50 - 25\left(\frac{-18}{155}\right) = 63.23$$

$$\lambda = 48.71 < 50 - 25\frac{M_1}{M_2} = 63.23$$

Therefore, second order effect can be neglected.

$$e_{total} = e_e + e_a = 121.23mm + 20mm = 141.23\ mm$$

$$M_{sd} = Pe_{Total} = 1280\ KN\left(\frac{141.23}{1000}\right) = 180.77KN\ m$$

$$\mu_{sd} = \frac{M_{sd}}{A_c f_{cd} h} = \frac{180.77 \times 10^6}{(300 \times 400) \times 13.6x400mm} = 0.28$$

$$\nu_{sd} = \frac{N_{sd}}{A_c f_{cd}} = \frac{1280 \times 10^3}{(300 \times 400) \times 13.6} = 0.78$$

- From uniaxial ,chart number 2

$$\omega = 0.6$$

- **Design consents:**

$$C-30, \quad f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} \quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S-400, \quad f_{yk} = 400\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{400}{1.15} = 347.83\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165MPa$$

- **Determine Reinforcement:**

$$A_s = \frac{\omega A_c f_{cd}}{f_{cd}} = \frac{0.6 \times (300 \times 400) \times 13.6}{347.83} = 2815.2mm^2$$

$$\Rightarrow A_s = 2815\ mm^2 (use\ 8\Phi_{22} = 3040mm^2)$$

- ***Check*** $:\boldsymbol{A_{s,min} \leq A_s \leq A_{s,max}}$
- **The minimum reinforcement:**

$$A_{s,min} = 0.008A_c = [0.008 * (300mm * 400m)] = 960mm^2$$

$$A_{s,max} = 0.08A_c = [0.08 * (300mm * 400m)] = 9600mm^2$$

$$A_{s,min} \leq A_{s,proved} \leq A_{s,\max}$$

$$\rightarrow 960mm^2 < 3040\ mm^2 \leq 9600mm^2$$

Therefore, it is ok!

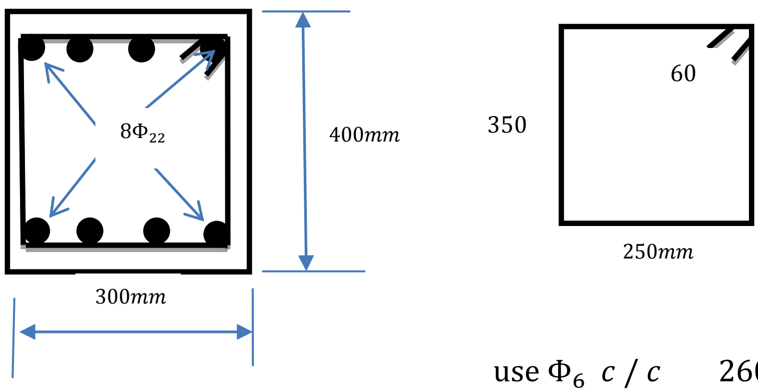
- **Lateral Reinforcement:**

$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{22}{4} = 5.5mm \end{cases} \qquad \Rightarrow \Phi = 10\ mm$$

$$S \leq \begin{cases} 300\ mm \\ b = 300\ mm \\ 12\Phi = 12x22mm = 264mm \end{cases} \qquad \Rightarrow S = 260mm$$

$$\text{use } \Phi_6 \quad c/c \ \ 260mm$$

- **Sketch:**



use Φ_6 c/c $260mm$ $L = 1320mm$

3.8 Design of Column under Bi-axial bending

There are situations in which axial compression is accompanied by simultaneous bending about both principal axes of the section. This is the case in corner columns, interior or edge columns with irregular column layout. For such columns, the determination of failure load is extremely laborious and making manual computation difficult.

Consider the Reinforced concrete column section shown under axial force p acting with eccentricities e_x and e_y, such that $e_x = M_y/P$, $e_y = M_x/P$ from Centriodal axes (Fig. 3.13c).

In Fig. 3.13a the section is subjected to bending about the y axis only with eccentricity e_x. The corresponding strength interaction curve is shown as Case (a) (see Fig. 3.13d). Such a curve can be established by the usual methods for uni-axial bending. Similarly, in Fig. 3.13b the section is subjected to bending about the x axis only with eccentricity e_y. The corresponding strength interaction curve is shown as Case (b) (see Fig. 3.13d). For case (c), which combines x and y axis bending, the orientation of the resultant eccentricity is defined by the angle λ:

$$\lambda = \arctan\frac{e_x}{e_y} = \arctan\frac{M_{ny}}{M_{nx}}$$

Bending for this case is about an axis defined by the angle θ with respect to the x-axis. For other values of λ, similar curves are obtained to define the failure surface for axial load plus bi-axial bending.

Any combination of P_u, M_{ux}, and M_{uy} falling outside the surface would represent failure. Note that the failure surface can be described either by a set of curves defined by radial planes passing through the P_n axis or by a set of curves defined by horizontal plane intersections, each for a constant P_n, defining the load contours (see Fig. 3.13d).

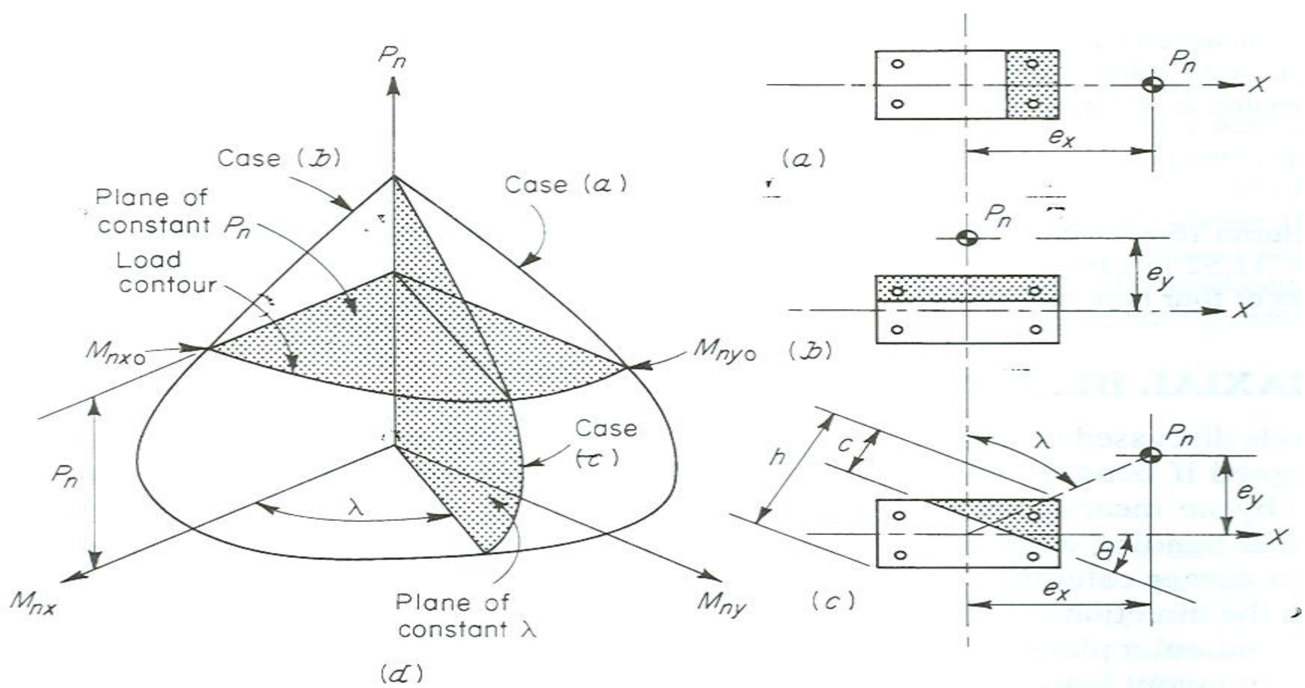


Fig.3.13 Interaction diagram for compression plus bi-axial bending

Computation commences with the successive choice of neutral axis distance c for each value of q. Then using the strain compatibility and stress-strain relationship, bar forces and the concrete compressive resultant can be determined. Then P_n, M_{nx}, and M_{ny}, (a point on the interaction surface) can be determined using the equation of equilibrium (see below).

$$\Sigma F_h = 0 \quad \Rightarrow \quad P_n = P_c + P_{sc} + P_{st} \qquad (1^*)$$

Where:

$P_c = \Sigma A_{ci} f_i$, resultant force in concrete

$P_{sc} = \Sigma A_{sci} f_{sci}$, resultant compressive force in compression steel

$P_{st} = \Sigma A_{sti} f_{sti}$, resultant tensile force in tension steel

$$\Sigma M_{nx} = \Sigma A_{ci} f_{ci} y_{ci} + \Sigma A_{sci} f_{sci} y_{sci} + \Sigma A_{sti} f_{sti} y_{sti} \qquad (2^*)$$

$$\Sigma M_{ny} = \Sigma A_{ci} f_{ci} x_{ci} + \Sigma A_{sci} f_{sci} x_{sci} + \Sigma A_{sti} f_{sti} x_{sti} \qquad (3^*)$$

Since the determination of the neutral axis requires several trials, the procedure using the above expressions is tedious. Thus, the following simple approximate methods are widely used.

i. **Load contour method**: It is an approximation on load versus moment interaction surface (see Fig. 3.13). Accordingly, the general non-dimensional interaction equation of family of load contours is given by:

$$\left(\frac{M_{dx}}{M_{dxo}}\right)^{\alpha_n} + \left(\frac{M_{dy}}{M_{dyo}}\right)^{\alpha_n} = 1$$

$$\alpha_n = 0.667 + 1.667\left(\frac{p_{da}}{p_{do}}\right) \quad \text{and} \quad 1.15 \le \alpha_n \le 2.0$$

Where:

- $M_{dx} = P_d e_y$
- $M_{dy} = P_d e_x$
- $M_{dxo} = M_{dx}\ when\ M_{dy} = 0\ (design\ capacity\ under\ uni-axial\ bending\ about\ x)$
- $M_{dyo} = M_{dy}\ when\ M_{dx} = 0\ (design\ capacity\ under\ uni-axial\ bending\ about\ y)$

ii. **Reciprocal method/Bresler's equation**: It is an approximation of bowl shaped failure surface by the following reciprocal load interaction equation.

$$\frac{1}{p_{dx}} = \frac{1}{p_{dxo}} + \frac{1}{p_{dyo}} - \frac{1}{p_{do}}$$

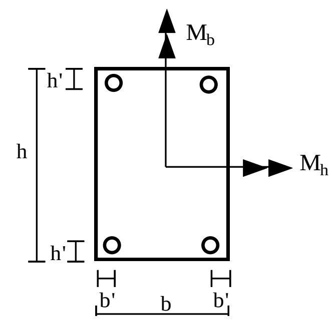
Where:

- P_d = design (ultimate) load capacity of the section with eccentricities e_{dy} & e_{dx}

- P_{dxo} = ultimate load capacity of the section for uni axial bending with e_{dx} only $(e_{dy} = 0)$
- P_{dyo} = ultimate load capacity of the section for uni axial bending with e_{dy} only $(e_{dx} = 0)$
- P_{do} = concentric axial load capacity $(e_{dyx} = e_{dy} = 0)$

However interaction charts prepared for biaxial bending can be used for actual design. The procedure involves:

- Select cross section dimensions h and b and also h' and b'
- Calculate $\frac{h'}{h}$ and $\frac{b'}{b}$ (range of values of 0.0, 0.2, 0.4 …, 1.4 are available)
- Compute:
  - Normal force ratio: $\upsilon = \dfrac{N}{f_{cd}A_c}$
  - Moment ratios:



$$\mu_h = \frac{M_h}{f_{cd}A_c h}, \quad \mu_b = \frac{M_b}{f_{cd}A_c b}$$

- Select suitable chart which satisfy $\frac{h'}{h}$ and $\frac{b'}{b}$ ratio:
- Enter the chart to obtain ω
- Compute $A_{s,tot} = \frac{\omega A_c f_{cd}}{f_{yd}}$
- Check A_{tot}, satisfies the maximum and minimum provisions
- Determine the distribution of bars in accordance with the charts requirement

Example 3.8.1

Design a column to sustain a factored design axial load of 900 KN and biaxial moments of$M_{dx} = 270\ KNm$, $M_{dy} = 180\ KN\ m$, including all other effects. Assume concrete C-30, steel S-300 class –I work. Assume ${}^{b}/_{h} = {}^{400}/_{600mm}$

Solution:

- **Design constant:**

$$C-30, \quad f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} \quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S-300, \quad f_{yk} = 300\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{300}{1.15} = 260.87\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165MPa$$

$$Assume\ \frac{b'}{b} = \frac{h'}{h} = 0.1$$

$$P_d = N_{sd} = 900\ KN$$

$$M_h = M_{dx} = 270\ KNm$$

$$M_b = M_{dy} = 180\ KNm$$

$$\nu_{sd} = \frac{N_{sd}}{A_c f_{cd}} = \frac{900\ KN}{(400x600) * 13.6} = 0.28$$

$$\mu_h = \frac{M_h}{A_c f_{cd} h} = \frac{270 \times 10^6}{(400 \times 600) \times 13.6 * 600} = 0.14$$

$$\mu_b = \frac{M_b}{A_c f_{cd} b} = \frac{180 \times 10^6}{(400 \times 600) \times 13.6 * 400} = 0.14$$

$$since\ \nu_{sd} = 0.28\ (\ between\ 0.2\ \&\ 0.4\ )$$

Therefore, interpolate to obtain ω

Using biaxial chart Number 9

$$\left.\begin{matrix} \nu = 0.2 \\ \mu_b = 0.14 \\ \mu_h = 0.14 \end{matrix}\right\} \Rightarrow \omega_1 = 0.4 \quad , \quad \left.\begin{matrix} \nu = 0.4 \\ \mu_b = 0.14 \\ \mu_h = 0.14 \end{matrix}\right\} \Rightarrow \omega_2 = 0.4$$

By interpolate for $\nu = 0.28$ $\quad we\ get\ \omega = 0.4$

- **Determine Reinforcement:**

$$A_s = \frac{\omega A_c f_{cd}}{f_{cd}} = \frac{0.4 \times (400 \times 600) \times 13.6}{260.87} = 5005\ mm^2$$

$$\Rightarrow A_s = 5005\ mm^2 (use\ 8\Phi_{30} = 5655\ mm^2)$$

- **Check :**$\boldsymbol{A_{s,min} \leq A_{s,proved} \leq A_{s,max}}$
- **he minimum reinforcement:**

$$A_{s,min} = 0.008A_c = [0.008 * (400mm \times 600m)] = 1920mm^2$$

$$A_{s,max} = 0.08A_c = [0.08 * (400mm \times 600m)] = 19200mm^2$$

$$A_{s,min} \leq A_{s,proved} \leq A_{s,\max}$$

$$\Rightarrow 1920mm^2 < 5005\ mm^2 \leq 19200mm^2 \quad it\ is\ ok!$$

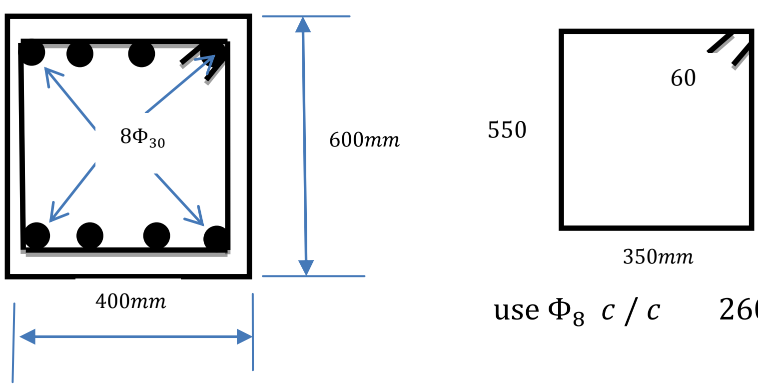
- **Lateral Reinforcement:**

$$\Phi \geq \begin{cases} 6mm \\ \frac{\Phi}{4} = \frac{30}{4} = 7.5mm = 8mm \end{cases} \qquad \Rightarrow \Phi = 8\,mm$$

$$S \leq \begin{cases} 300\,mm \\ b = 400\,mm \\ 12\Phi = 12x30mm = 360\,mm \end{cases} \qquad \Rightarrow S = 300mm$$

$$\text{use } \Phi_8 \;\; c/\,c \;\; 300mm$$

- **Sketch:**



use Φ_8 $c\,/\,c$ $260mm$ $L = 1920mm$

Example 3.8.2

A square column subjected to a design load of 1000KN and constant first order moment of $M_{dx} = 98\,KN\,m$ and $M_{dy} = 87KNm$. If martial's C-25 and S-300 class I works are used and second order eccentricity is approximated as 10% of the constant first-order value($e_2 = 0.1e_e$) in both directions and $e_{ax} = e_{ay} = 20mm$. Assume the column dimension $^{b}/_{h} = {}^{450}/_{450mm}$.

Design this column using:

a) Biaxial chart number 1 & 2
b) Bresler's reciprocal load equation

Solution:

a. Biaxial chart

- **Design constant:**

$$C-25, \quad f_{cd}=\frac{0.85f_{ck}}{\gamma_c} \quad ,f_{ck}=\frac{f_{cu}}{1.25}=\frac{25}{1.25}=20Mpa$$

$$\Rightarrow f_{cd}=\frac{0.85f_{ck}}{\gamma_c}=0.85*\frac{20}{1.5}=11.33MPa$$

$$S-300, \quad f_{yk}=300\ Mpa$$

$$f_{yd}=\frac{f_{yk}}{\gamma_s}=\frac{300}{1.15}=260.87\ MPa$$

$$f_{ctd}=\frac{0.21f_{ck}^{\frac{2}{3}}}{\gamma_c}=\frac{0.21*20^{\frac{2}{3}}}{1.5}=1.0315MPa$$

$$Let\ \frac{b'}{b}=\frac{h'}{h}=0.1$$

$$M_{dx}=P_d e_{x,total}=P_d(e_{ex}+e_{ax}+e_{2x})$$
$$=P_d(e_{ex}+0.1e_{ex}+20)$$
$$=1000(1.1e_{ex}+20)$$
$$=1.1M_x+1000*\left(\frac{20}{1000}\right)$$
$$\Rightarrow M_{dx}=M_h=(1.1*98)+20=127.8\ KN\ m$$

$$M_{dy}=P_d e_{y,total}=P_d(e_{ey}+e_{ay}+e_{2y})$$
$$=P_d\left(e_{ey}+0.1e_{ey}+20\right)$$
$$=1000\left(1.1e_{ey}+20\right)$$
$$=1.1M_y+1000*\left(\frac{20}{1000}\right)$$
$$\Rightarrow M_{dy}=M_b=(1.1*87)+20=115.7\ KN\ m$$

Using biaxial chart number 1 & 2

$$\nu_{sd}=\frac{N_{sd}}{A_c f_{cd}}=\frac{1000\times 10^3\ N}{(450^2)*11.33}=0.44$$

$$\mu_h=\frac{M_h}{A_c f_{cd} h}=\frac{127.8\times 10^6}{(450^2)*11.33*450}=0.12$$

$$\mu_b=\frac{M_b}{A_c f_{cd} b}=\frac{115.7\times 10^6}{(450^2)*11.33*450}=0.11$$

Using biaxial chart Number 2

$$\left.\begin{array}{r}\nu = 0.4\\ \mu_b = 0.11\\ \mu_h = 0.12\end{array}\right\} \Rightarrow \omega_1 = 0.2 \quad , \quad \left.\begin{array}{r}\nu = 0.6\\ \mu_b = 0.11\\ \mu_h = 0.12\end{array}\right\} \Rightarrow \omega_2 = 0.22$$

By interpolation for $\nu = 0.44 \quad we\ get\ \omega = 0.204$

- **Determine Reinforcement:**

$$A_{s,tot} = \frac{\omega A_c f_{cd}}{f_{cd}} = \frac{0.204 \times (450 \times 450) \times 11.33}{260.87} = 1794\ mm^2$$

$$\Rightarrow A_s = 1794\ mm^2 (use\ 4\Phi_{24} = 1810\ mm^2)$$

- **Check :$A_{s,min} \le A_{s,proved} \le A_{s,max}$**
- **he minimum reinforcement:**

$$A_{s,min} = 0.008 A_c = [0.008 * (450mm \times 450m)] = 1620mm^2$$
$$A_{s,max} = 0.08 A_c = [0.08 * (450mm \times 450m)] = 16200mm^2$$
$$A_{s,min} \le A_{s,proved} \le A_{s,\max}$$
$$\Rightarrow 1620mm^2 < 1810mm^2 \le 16200mm^2 \quad it\ is\ ok!$$

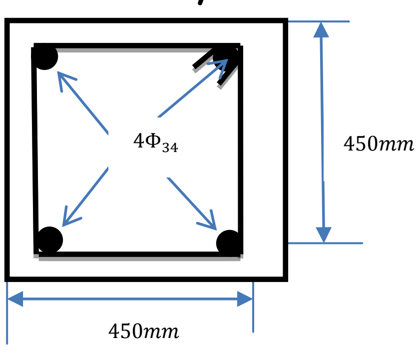
- **Lateral Reinforcement:**

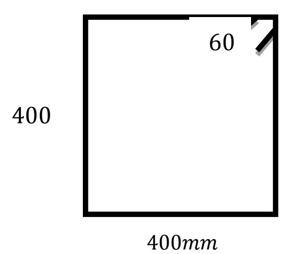
$$\Phi \ge \begin{cases} 6mm \\ \dfrac{\Phi}{4} = \dfrac{24}{4} = 6mm \end{cases} \qquad \Rightarrow \Phi = 6\ mm$$

$$\mathrm{S} \le \begin{cases} 300\ mm \\ b = 450\ mm \\ 12\Phi = 12x24mm = 288\ mm \end{cases} \qquad \Rightarrow S = 288mm$$

$$\text{use } \Phi_8 \quad c/c \quad 280mm$$

- **Reinforcement Layout:**





$$\text{use } \Phi_6 \ \ c\,/\,c \qquad 280mm \qquad L = \ 1720mm$$

b. Using Bresler's reciprocal load equation

$$P_d = 1000KN$$

$$Assume\ ^{b}/_{h} = ^{450}/_{450}\ \& A_{st} = 4\Phi_{24} = 1810mm^2$$

$$\omega = \frac{A_s f_{yd}}{A_c f_{cd}} = \frac{1810 \times 260.87}{450^2 \times 11.33} = 0.21$$

- **Section capacity (axial load)**

$$P_{d0} = P_{concrete} + P_{steel}$$

$$= f_{cd}(A_c - A_{st}) + f_{yd}A_{st}$$

$$= 11.33[(450^2) - 1810] + (260.87 \times 1810)$$

$$= 2745.99\ KN$$

From uniaxial chart number 2

$$P_{dnx}: \quad \left.\begin{matrix} \mu_b = 0.11 \\ \omega = 0.21` \end{matrix}\right\} \Rightarrow \nu = 0.91$$

$$\nu = 0.91 = \frac{P_{dnx}}{A_c f_{cd}} \quad \Rightarrow\ \mathrm{P_{dnx}} = 0.91 * 11.33 * 450^2 = 2088\ \mathrm{KN}$$

$$P_{dny}: \quad \left.\begin{matrix} \mu_H = 0.12 \\ \omega = 0.21` \end{matrix}\right\} \Rightarrow \nu = 0.88$$

$$\nu = 0.88 = \frac{P_{dnx}}{A_c f_{cd}} \quad \Rightarrow\ \mathrm{P_{dnx}} = 0.88 * 11.33 * 450^2 = 2019\ \mathrm{KN}$$

$$P_{dn} = \frac{P_{do}P_{dnx}P_{dny}}{P_{do}\left(P_{dnx} + P_{dny}\right) - \left(P_{dnx}P_{dny}\right)}$$

$$= \frac{(2745.99 \times 2088 \times 2019)}{2745.99(2088 + 2019) - (\ 2088 \times 2019)}$$

$$P_{dn} = 1639\ KN \geq 1000KN\ , it\ is\ ok!$$

Example 3.8.3

Design a biaxial column subjected to a design load of $1525KN$ and constant first order moment of $M_{dx} = 125\ KNm\ \ and\ M_{dy} = 175\ KN\ m$ assume the column is fixed at bottom end and free to rotate at the top end in both direction use$C - 30$, $S - 460$, class I –works. Take height column to be $6.2m$ and $b/D = 400/400$. Neglect the 2^{nd} order effect.

Solution:

Givens:

$$P_d = N_{sd} = 1525\ KN$$
$$M_{dx} = 125\ KNm$$
$$M_{dy} = 175\ KNm$$

- The column is fixed at the bottom and free at the top end, hence

$$L_e = 2L = 2 * 6.2m = 12.4m$$

- The material are:

$$Concrete : C - 30$$
$$Steel: S - 460$$

- $Height\ of\ columnis\ \ 6.2m$
- ${}^{b}/_{h} = {}^{400}/_{400}$

➢ **Design constant:**

$$C - 30,\quad f_{cd} = \frac{0.85 f_{ck}}{\gamma_c}\quad , f_{ck} = \frac{f_{cu}}{1.25} = \frac{30}{1.25} = 24Mpa$$

$$\Rightarrow f_{cd} = \frac{0.85 f_{ck}}{\gamma_c} = 0.85 * \frac{24}{1.5} = 13.6MPa$$

$$S - 400,\quad f_{yk} = 400\ Mpa$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{400}{1.15} = 347.83\ MPa$$

$$f_{ctd} = \frac{0.21 f_{ck}^{\frac{2}{3}}}{\gamma_c} = \frac{0.21 * 24^{\frac{2}{3}}}{1.5} = 1.165MPa$$

$$Assume\ d' = 20 + 10 + \frac{20}{2} = 40mm$$

$$d'/_h = \frac{40}{400} = 0.1$$

- **Determine the eccentricities:**

$$e_{total} = e_a + e_e + e_2$$

$$e_a \geq \begin{cases} \frac{L_e}{300} = \frac{1240}{300} = 41mm \\ 20mm \end{cases}$$

$$e_a = 41mm - additionentrial\ eccentricity$$

$$e_{total,x} = e_a + e_{ex}$$

$$e_{total,y} = e_a + e_{ey}$$

$$M_{sd,x} = P_d\ e_{tatal,x}$$

$$M_{sd,x} = P_d\ e_{e,x} + P_d\ e_a$$

$$M_{sd,x} = M_{dx} + P_d\ e_a$$

$$\Rightarrow M_{sd,x} = 125\ KNm + 1525KN * \frac{41}{1000}m = 187.5\ KNm$$

$$\Rightarrow M_{sdh} = M_{sd,x} = 187.5\ KN\ m$$

$$M_{sd,y} = P_d\ e_{tatal,y}$$

$$M_{sd,y} = P_d\ e_{e,y} + P_d\ e_a$$

$$M_{sdy,} = M_{dy} + P_d\ e_a$$

$$\Rightarrow M_{sdy} = 175\ KNm + 1525KN * \frac{41}{1000}m = 237.5\ KNm$$

$$\Rightarrow M_{sdb} = M_{sdy} = 237.5\ KN\ m$$

$$\nu_{sd} = \frac{N_{sd}}{A_c f_{cd}} = \frac{1525KN}{(400^2) * 13.6} = 0.7$$

$$\mu_h = \mu_{sdh} = \frac{M_{sdh}}{A_c f_{cd} h} = \frac{187.5KNm}{(400^2) * 13.6 * 400} = 0.21$$

$$\mu_b = \mu_{sdb} = \frac{M_{sdb}}{A_c f_{cd} b} = \frac{237.5KNm}{(400^2) * 13.6 * 400} = 0.27$$

$$since\ \nu_{sd} = 0.7\ (\ between\ 0.6\ \&\ 0.8\ )$$

Use inter pollutant together the value $is\ \omega$

By using chart number 5 & chart number 6

$$\upsilon = 0.6$$

$$\left.\begin{matrix} \mu_b = 0.27 \\ \mu_h = 0.21 \end{matrix}\right\} \Rightarrow \omega_1 = 0.95$$

$$\upsilon = 0.8$$

$$\left.\begin{matrix} \mu_b = 0.27 \\ \mu_h = 0.21 \end{matrix}\right\} \Rightarrow \omega_2 = 1$$

Using inter pollution for $\upsilon = 0.7$

υ	ω
0.6	0.95
0.7	x
0.8	1.0

$$x = 0.975 = \omega\ \ for\ \ \upsilon = 0.7$$

$$A_{s,total} \frac{\omega A_C f_{cd}}{f_{yd}} = \frac{0.975x(400x400)x13.6}{400} = 5304mm^2$$

$$\text{use } 6\Phi_{34}\ (A_{s,provid} = 5448mm^2\ )$$

- **Check** $A_{s,min} = 0.008\,x(400mmx400mm) = 1280mm^2$

$$A_{s,max} = 0.08\,x(400mmx400mm) = 12{,}800mm^2$$

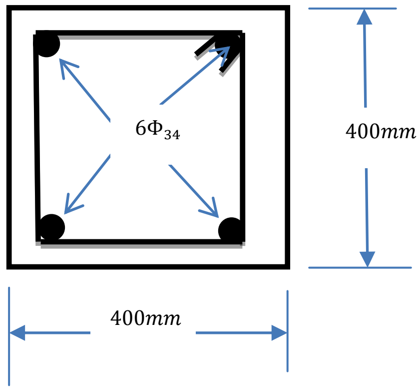
Since $A_s min \leq A_s, proved\ \leq A_{s\ max}$ $\quad its\ ok!$

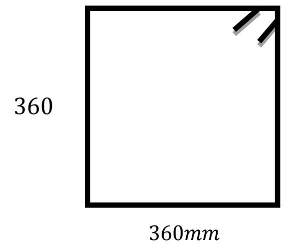
- **Determine the lateral reinforcement**:

$$\Phi \geq \begin{cases} \dfrac{\phi}{4} = \dfrac{34}{4} = 8.5 \\ 6 \end{cases}, \qquad S \leq \begin{cases} 300mm \\ 12\Phi = 12x34 = 408mm \\ b = 400mm \end{cases}$$

$$\text{use } \Phi_{10}\ \ c\ /\ c\ \ 300mm$$

- **Reinforcement Layout:**





$\text{use } \Phi_{10} \;\; c\,/\,c \qquad 300mm \qquad L = 1560mm$

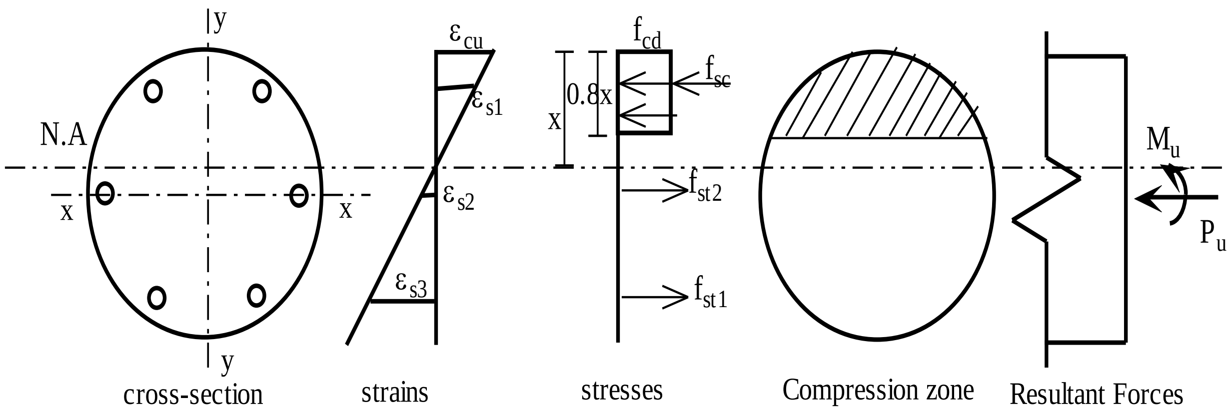
3.9 Circular columns

When load eccentricities are small, spirally reinforced columns show greater ductility (greater toughness) than tied columns. However the difference fades out as the eccentricity is increased.

Interaction Diagram for Circular columns

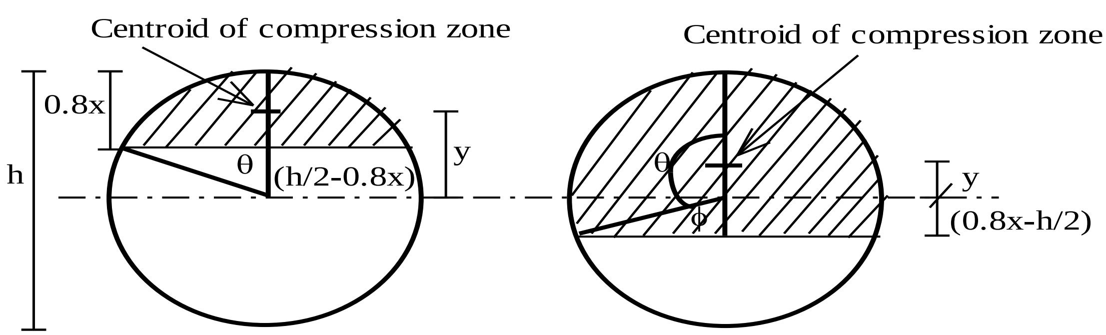
The strain compatibility solution described in the preceding section can also be used to calculate the points on an interaction diagram for circular columns

Consider the following circular cross section reinforced with 6 longitudinal bars



Calculations can be carried out in the same way as in the previous section except that for circular columns the concrete compression zone subject to the equivalent rectangular stress distribution has the shape of a segment of a circle (see above).

To compute the compressive force and its moment about the centroid of the column, it is necessary to be able to compute the area and centroid of the segment.



Case 1 : $0.8x \leq h/2$, $\theta < 90°$ Case 2 : $0.8x > h/2$, $\theta > 90°$

$$\theta = \cos^{-1}\left[\frac{h/2 - 0.8x}{h/2}\right] \qquad \phi = \cos^{-1}\left[\frac{0.8x - h/2}{h/2}\right],\ \theta = 180° - \phi$$

The area of the segment is:

$$A = h^2\left(\frac{\theta - \sin\theta\cos\theta}{4}\right)$$

The moment of this area about the centre of the column is:

$$A\,\bar{y} = h^3\left(\frac{\sin^3\theta}{12}\right)$$

Where θ is expressed in radians.

The shape of interaction diagram of a circular column is affected by the number of bars and their orientation relative to the direction of the neutral axis. Thus the moment capacity about axis x-x (see above) is less than that about axis y-y.

Since the designer has little control over the arrangement of bars in a circular column, the interaction diagram should be computed using the least favorable bar orientation. But for circular columns with more than 8 bars, this problem vanishes as the bar placement approaches a continuous ring.

Design or analysis of spirally reinforced columns is usually carried out by means of design aids.

Effect of Creep

Creep effects may be ignored if the increase in the first-order bending moments due to creep deformation and longitudinal force does not exceed 10%.

The effect of creep can be accounted by:

a) For isolated columns in non-sway structures, creep may be allowed for by multiplying the curvature for short-term loads(see the expression of curvature in second order eccentricity) by $(1 + \beta_d)$, where β_d, is the ratio of dead load design moment to total design moment, always taken as positive.
b) For sway frames, the effective column stiffness may be divided by $(1 + \beta_d)$, where β_d, is as defined above.

Slender columns bent about the major axis

A slender column bent about the major axis may be treated as bi-axially loaded with initial eccentricity e_a acting about the minor axis

Biaxial Bending of Columns

a) Small Ratios of Relative Eccentricity

Columns of rectangular cross-section which are subjected to biaxial bending may be checked separately for uni-axial bending in each respective direction provided the relative eccentricities are such that $k \leq 0.2$; where k denotes the ratio of the smaller relative eccentricity to the larger relative eccentricity.

The relative eccentricity, for a given direction, is defined as the ratio of the total eccentricity, allowing for initial eccentricity and second-order effects in that direction, to the column width in the same direction.

b) Approximate Method

Columns of rectangular cross-section which are subjected to biaxial bending may be checked separately for uni-axial bending in each respective.

If the above condition is not satisfied, the following approximate method of calculation can be used, in the absence of more accurate methods.

For this approximate method, one-fourth of the total reinforcement must either be distributed along each face of the column or at each corner. The column shall be designed for uni-axial bending with the following equivalent uni-axial eccentricity of load, e_{eq} along the axis parallel to the larger relative eccentricity:

$$e_{eq} = e_{tot}(1 + k\alpha)$$

Where e_{tot} denotes the total eccentricity in the direction of the larger relative eccentricity

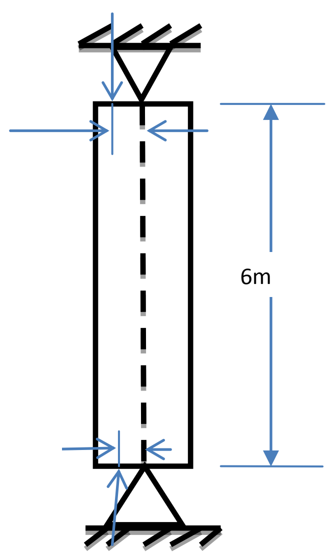
K denotes the relative eccentricity ratio as defined in above.

α may be obtained from the following table as a function of the relative normal force $\upsilon = \frac{N_{sdu}}{f_{cd} A_c}$

υ	0	0.2	0.4	0.6	0.8	≥ 1.0
α	0.6	0.8	0.9	0.7	0.6	0.5

Assignment-1

1. A pin ended columns as shown in figure 8 is to support an unfactored dead load of 400KN and an unfactored live load of 340KN. The load acts an eccentricity of 75mm at the top and 50mm at the bottom. Design this column by making use of charts if Martial's C-25, S-400(class I work) are used. Use column size of 400x400mm.



Exercises

2. A column sustains design moments $M_{dx} = M_{dy} = 60KNm$ in two orthogonal directions which include all effects in addition to an axial load of 700KN. If the material used are concrete C-25, S-400 class –I works. Determine the quantity reinforcement using biaxial chart number 1 and 2. Assume $^{b}/_{h} = {}^{300}/_{300mm}$
3. A rectangular column, 300x500mm is subjected to a factored design load of 1380KN and first order moment of$M_x = 135\ KN\ m\ and\ M_y = 60KNm$. If martials

C-25 and S-400 class I works are used and second order eccentricity is neglected in both directions and $L_{ex} = L_{ey} = 4m$.

a. Design this column using biaxial chart No. 9&10.
b. Verify your result in a using Bresler's reciprocal load equation

4. Determine the quantity of reinforcement required in slender braced column to resist a compressive force of 1400KN and first order moment of 145KNm and -20KNm. Assume b=h=300mm,length of column= $5.5m$,$L_e = 0.66L, f_{ck} = 35MPa$, & $f_{yk} = 460MPa$.
5. Constructed from materials C-30 and S-400 class I work. If the diagram for first order end moments and axial forces as obtained from analysis are shown in figure 12. Determine the quantity of reinforcement required. Assuming non sway frame system. Use $300mm/400mm$ & $L_e = 0.7L$. Shown the reinforcements details using a sketch a cross section.

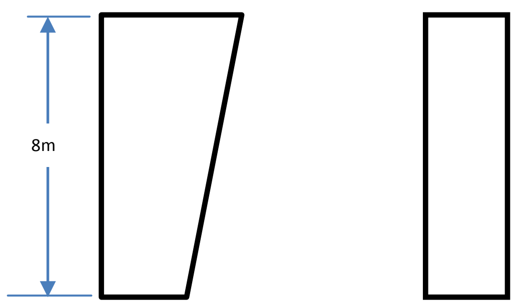
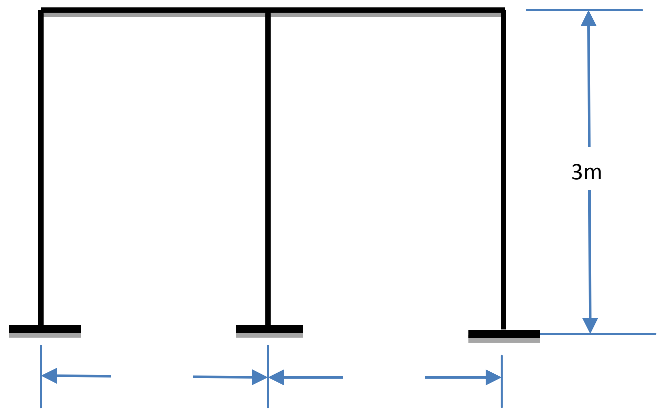
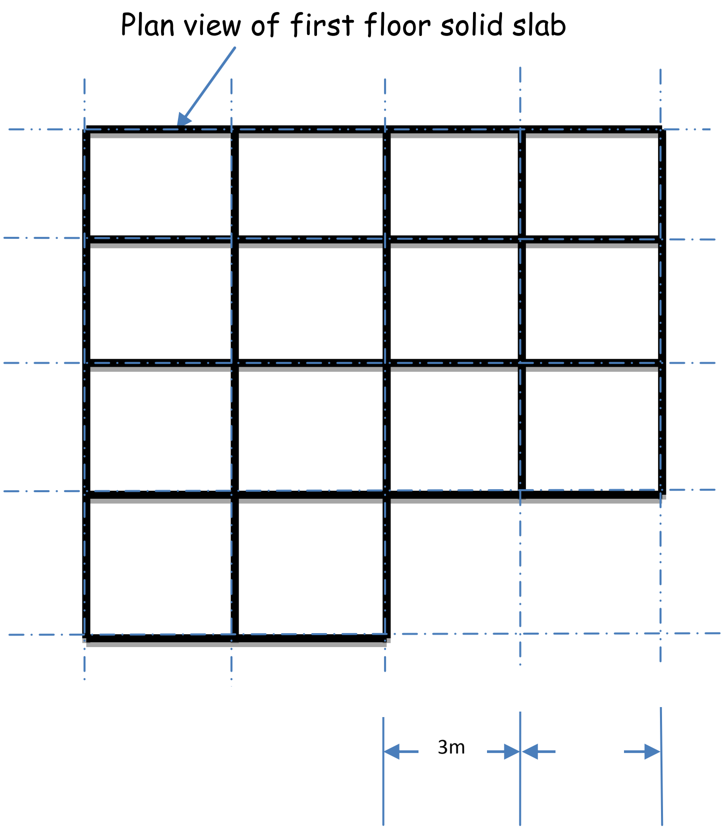
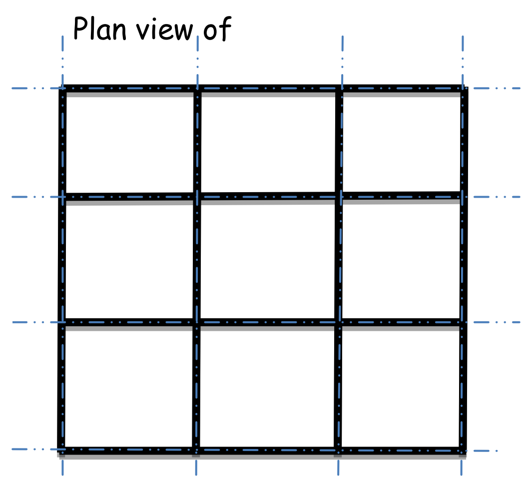


Figure 11

6. Suppose a total factored load of 50 KN/m including self-weight is transferred in to beams on axes A & 5 shown in figure 13, the moment of inertia of the beams have found to be twice that of the columns on that axes and for architectural reasons. The dimension of the columns is restricted to$500mm \times 500mm$. If materials $C-25, S-400$ class I works are used and assume non sway frame system with $L_e = 0.7L$
   i. Design column A_5 by meaning use of charts.
   ii. Verify your result in above by applying Bresler's reciprocal load equations.


3m


Plan view of first floor solid slab
3m


Plan view of

7. Figure 14 shown a frame supporting a total factored uniform load of 60 KN/m. using uniaxial chart number 2, design all the columns and show reinforcement details. Using sketch. If materials C-25,S-300,class I works are used. Assume non sway frame system with $L_e = 0.7L$ and note that for architectural reasons the dimension of the outer columns is restricted to $300mm \times 300mm$.

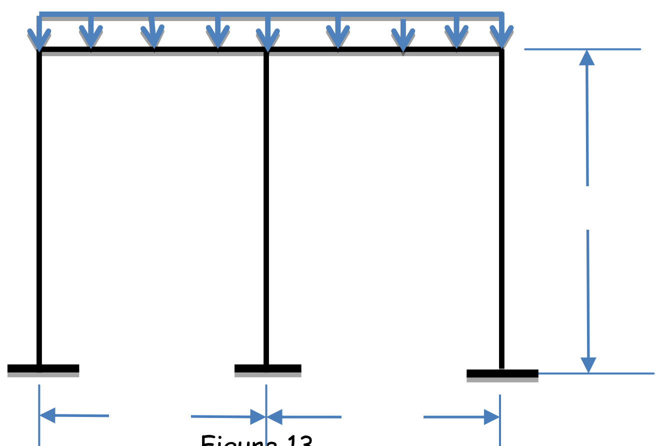
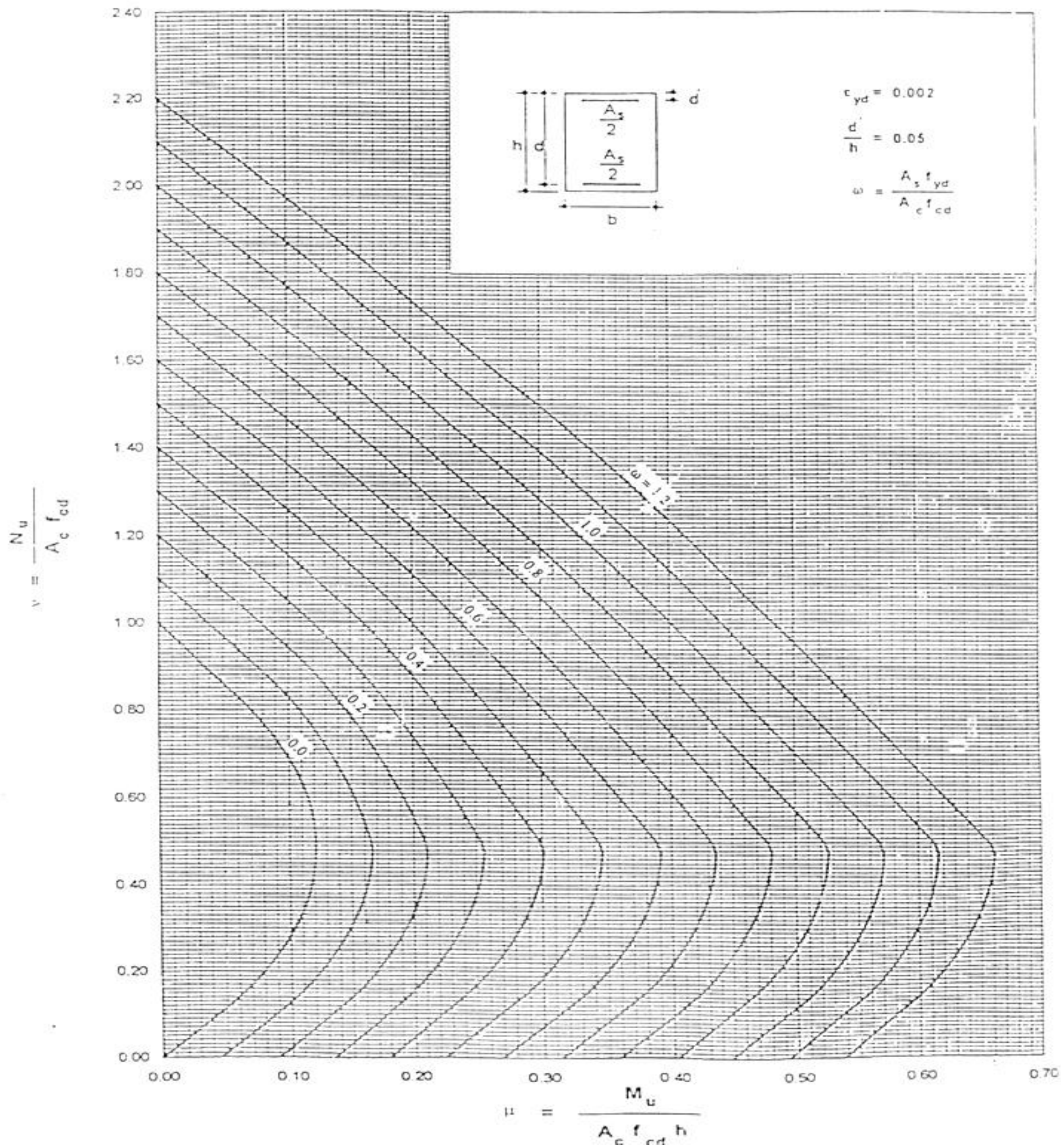
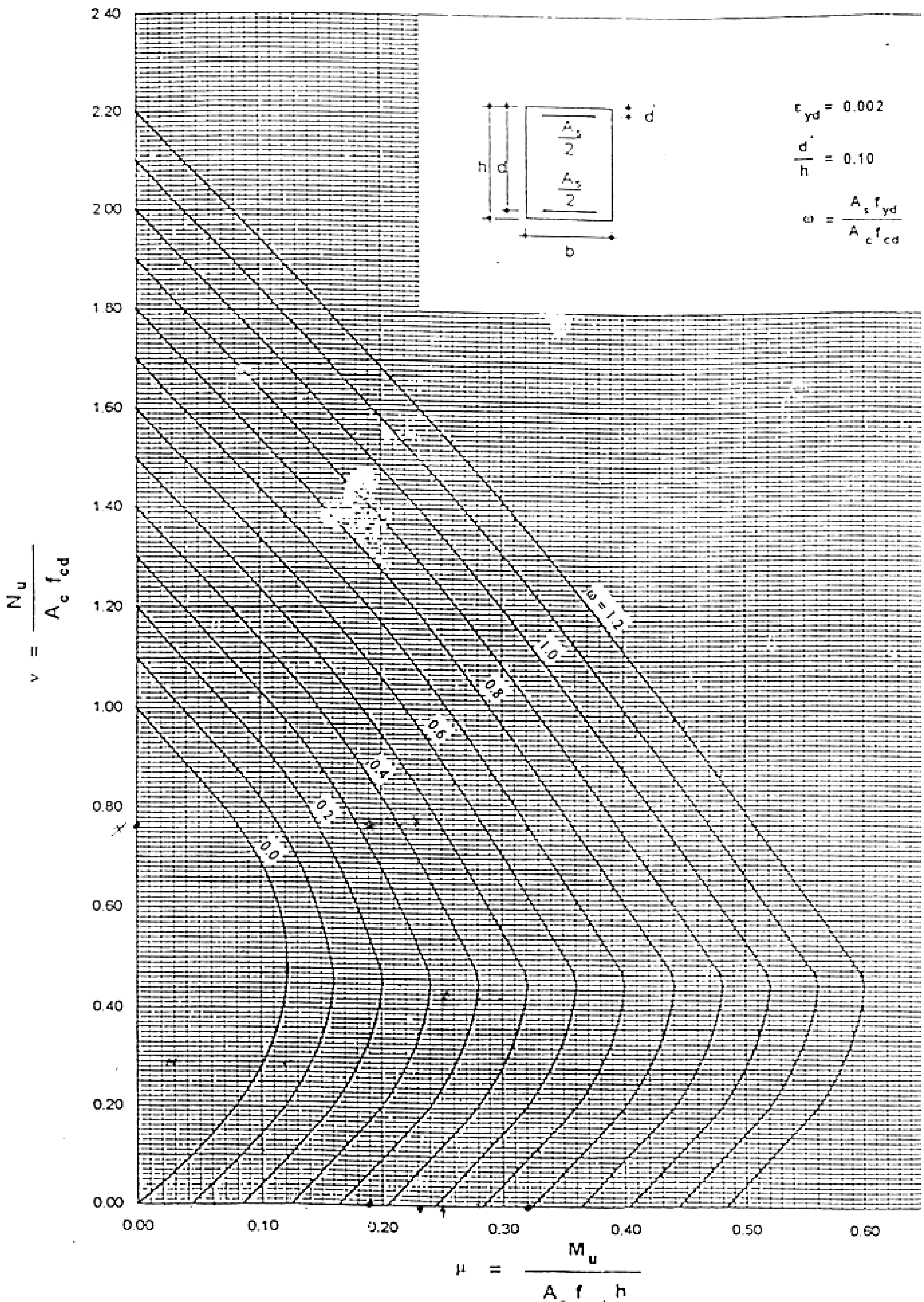


Figure 13

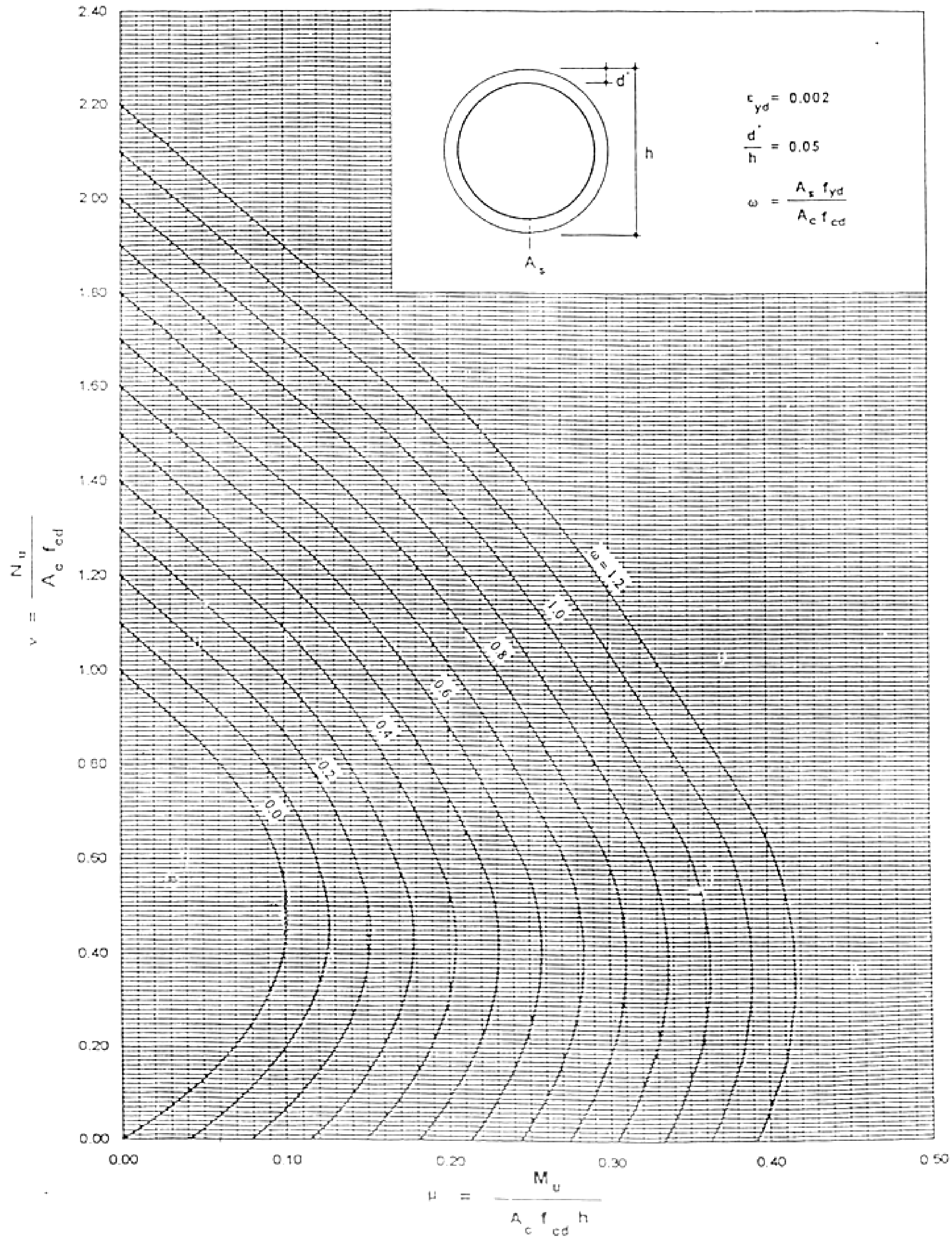
Uniaxial Chart No. 1



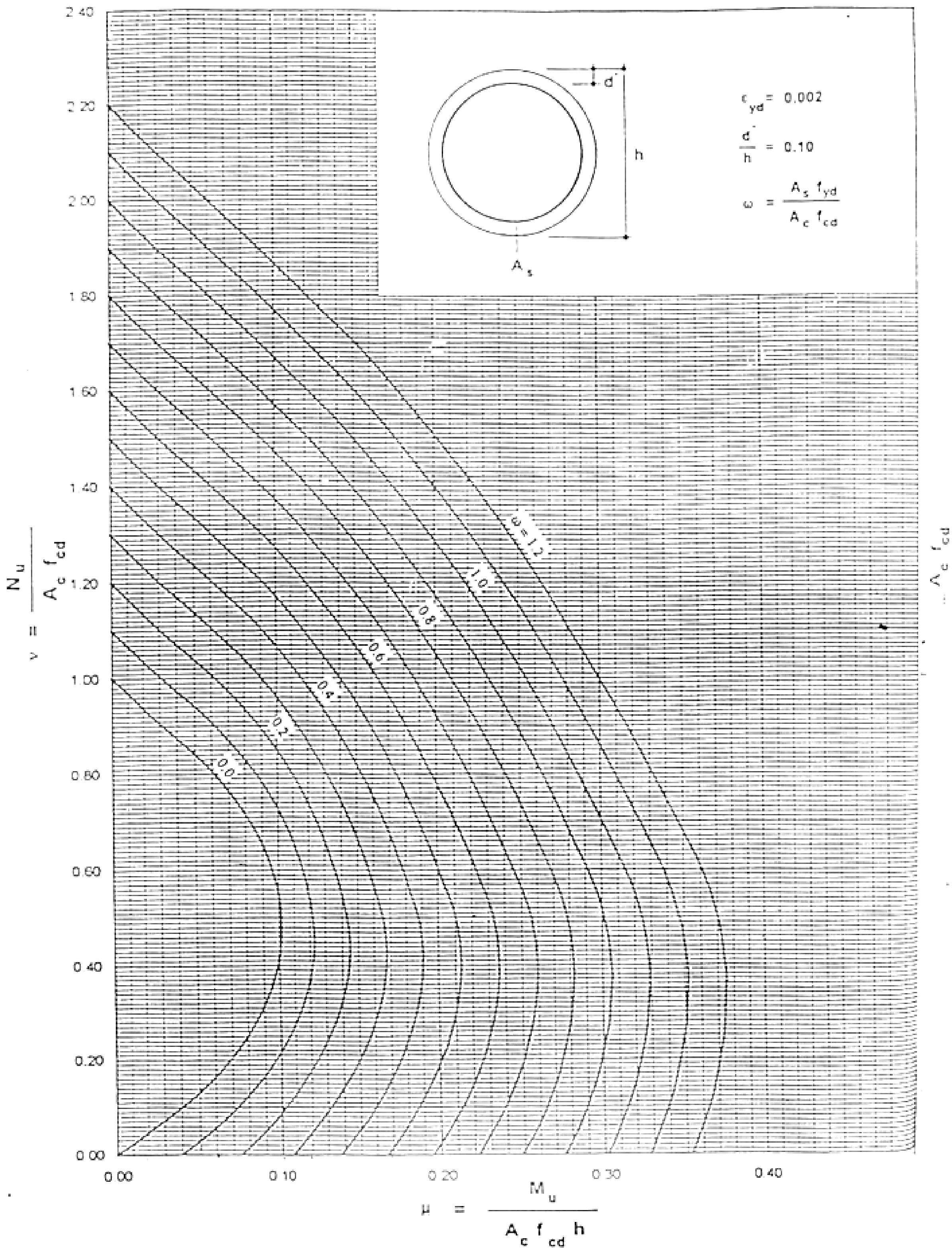
Uniaxial Chart No. 2



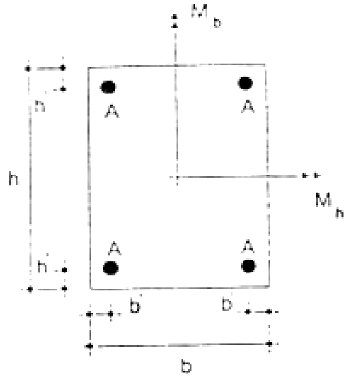
Uniaxial Chart No. 11



Uniaxial Chart No. 12



Biaxial chart No. 1



$\varepsilon_{yd} = 0{,}002$

$\frac{h'}{h} = \frac{b'}{b} = 0.10$

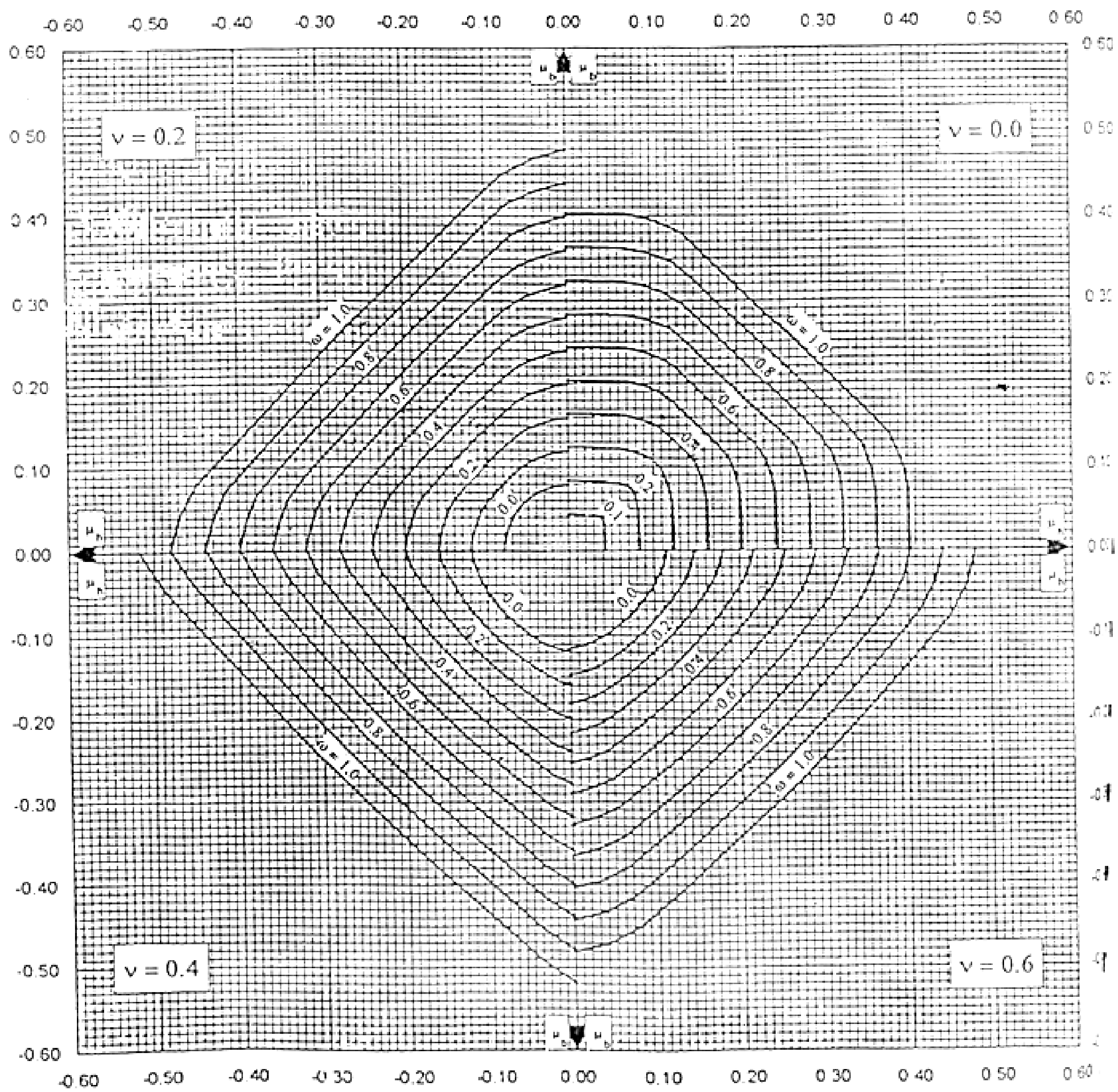
$A_{s,tot} = 4A$

$\omega = \frac{A_{s,tot} f_{yd}}{A_c \, f_{cd}}$

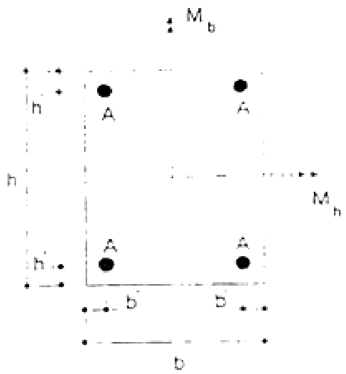
$\mu_h = \frac{M_h}{f_{cd} A_c h}$

$\mu_b = \frac{M_b}{f_{cd} A_c b}$

$\nu = \frac{N}{f_{cd} A_c}$



Biaxial Chart No. 2



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$$A_{s,tot} = 4A$$

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$$\mu_h = \frac{M_h}{f_{cd} A_c h}$$

$$\mu_b = \frac{M_b}{f_{cd} A_c b}$$

$$\nu = \frac{N}{f_{cd} A_c}$$